# Computer Exercise: TopoDrive

## Objectives

1. Review the concept of topography-driven flow.
2. Examine the effects of landform and geological heterogeneity.

## Groundwater flow equation

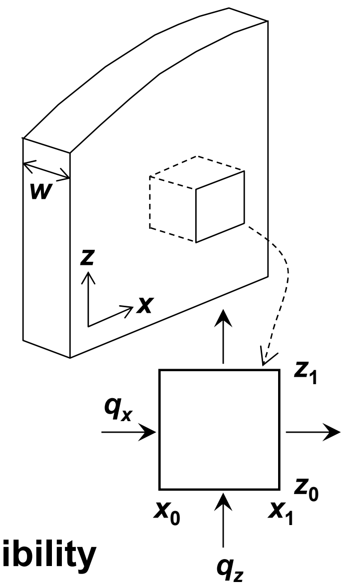
Slice of aquifer with thickness $w$ (m)

Net flow in $x$-direction ($m^3\ s^{-1}$):
= $[q_x(x_0) - q_x(x_1)] \times$ (cross-sectional area)

Net flow in $z$-direction ($m^3\ s^{-1}$):
= $[q_z(z_0) - q_z(z_1)] \times$ (cross-sectional area)

The rate of storage change in the box ($m^3\ s^{-1}$):
= $S_s \times \Delta h/\Delta t \times$ (box volume)

$S_s$ ($m^{-1}$): specific storage $\propto$ material compressibility






Mass balance of the box is given by:

$$(\text{net flow})_x + (\text{net flow})_z = \text{rate of storage change}$$

$$\frac{\partial}{\partial x}\left(K_x \frac{\partial h}{\partial x}\right) + \frac{\partial}{\partial z}\left(K_z \frac{\partial h}{\partial z}\right) = S_s \frac{\partial h}{\partial t}$$

The transient flow equation was 'applied' to GW problems by Theis (1935) and rigorously derived by Jacob (1940).

→ See a historical note by Bredehoeft (2008, *Hydrogeol. J.*, 16:5-9).

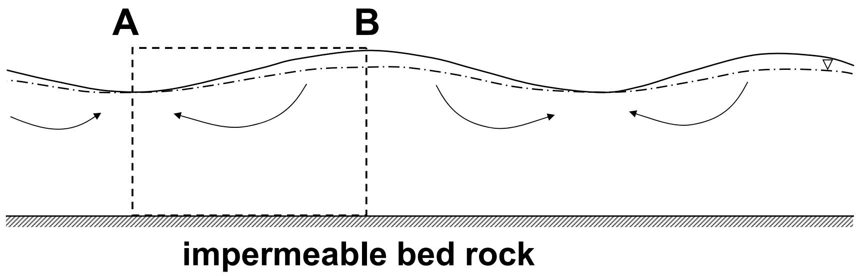
When 'average' flow is considered, the short term storage change becomes negligible → steady-state flow equation.

$$\frac{\partial}{\partial x}\left(K_x \frac{\partial h}{\partial x}\right) + \frac{\partial}{\partial z}\left(K_z \frac{\partial h}{\partial z}\right) = 0$$

This partial differential equation can be solved with appropriate boundary conditions to calculate hydraulic head distribution in a vertical cross section.

## Boundary conditions for flow equation

**Suppose a cross section of undulating terrain underlain by relatively impermeable bedrock. The shape of the water table resembles that of the land surface.**



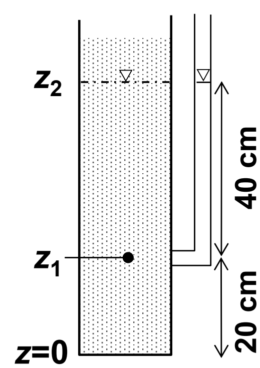

**Flow lines symmetrically converge at A and diverge at B.**
**→ A and B are considered impermeable boundaries.**

**The water table is commonly treated as the top boundary of the saturated region. To set a boundary condition at the water table, recall that:**

$$h = z + \psi$$



**In a column packed with sand, the water table is 60 cm above the bottom. Let's call the location of the water table Point 2.**




**Manometer at Point 1 indicates hydraulic head ($h$):**

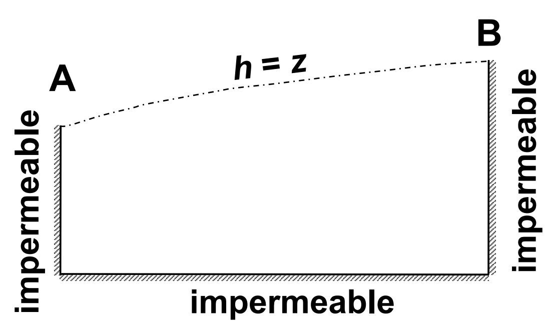
$$h_1 = z_1 + \psi_1 = 20\text{ cm} + 40\text{ cm} = 60\text{ cm}$$

**At Point 2, there is no standing water above:**

$$\rightarrow \psi_2 = 0 \qquad h_2 = z_2 + \psi_2 = 60\text{ cm} + 0$$

**At the water table $h = z$. The boundary value of $h$ is specified by elevation.**

**The boundary conditions shown in the figure will be used in the following exercises.**

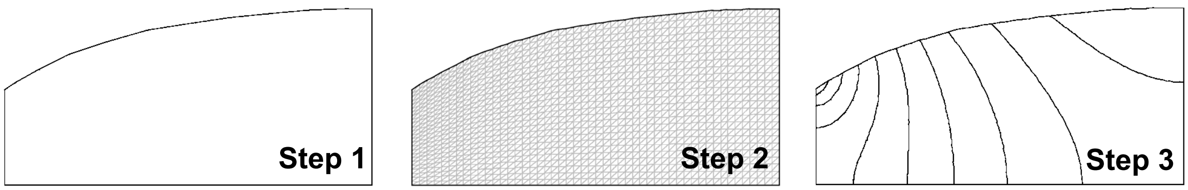


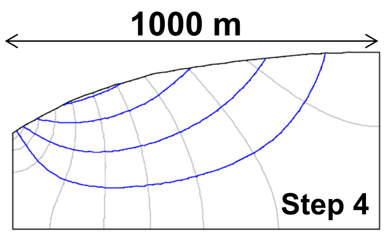
## TopoDrive program

**TopoDrive is a public-domain program written by Paul Hsieh at US Geological Survey (http://water.usgs.gov/nrp/gwsoftware/tdpf/tdpf.html).**

**Let's become familiar with the program by generating the famous Hubbert (1940, *J. Geol.*, 48: 785-944) section.**

**→ Step-by-step instruction on computers.**



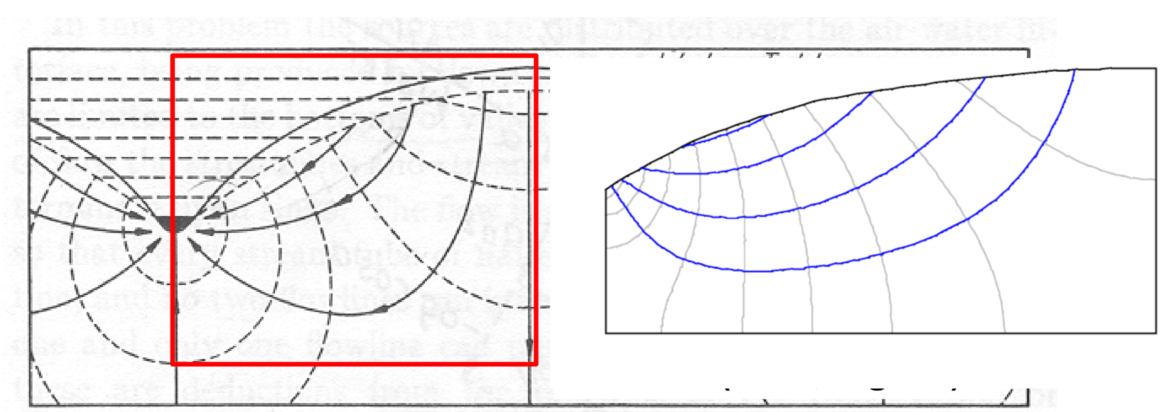




**Questions:**

**Where does groundwater recharge occur?**

**How about discharge?**





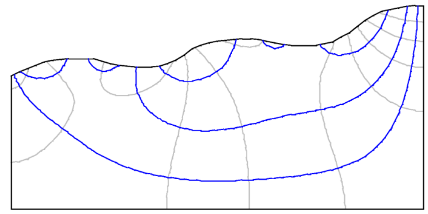
**Hubbert's concept was based on theoretical reasoning.**

**Mismatch was noted between the theory and field observation by Tóth (1962, *J. Geophys. Res.*, 67: 4375), who subsequently developed the rigorous mathematical treatment of topography-driven flow.**

**→ Read a fascinating story by Tóth (2002, *Ground Water*, 40: 320).**

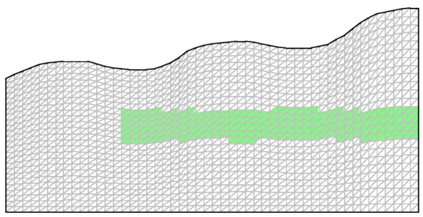
## Effects of undulating topography

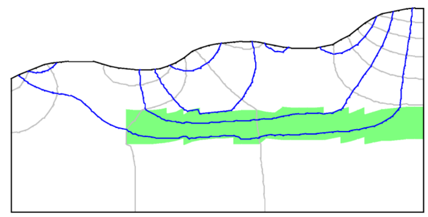
**Observations?**



**This concept was developed by Tóth (1963. *J. Geophys. Res.*, 68: 4795).**

## Effects of high-*K* layer





**Insert a layer with $K = 10^{-4}$ m/s**

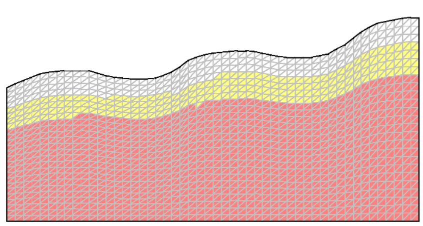
**Observations?**

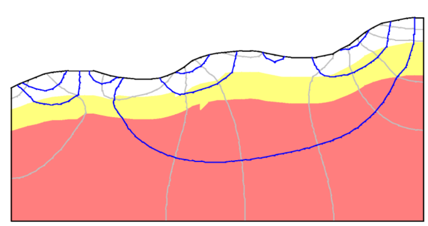
**This concept was studied by Freeze and Witherspoon (1967, *Water Resour. Res.*, 3: 623-634)**



## Effects of decreasing *K*

**Hydraulic conductivity of geological materials generally decreases with depth. This is related to the development of fractures and macropores near the surface or higher degree of compaction at depths.**





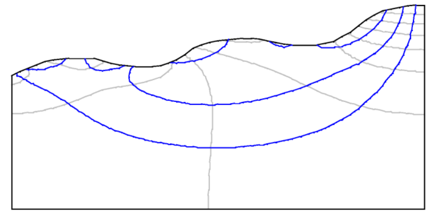
**Add a middle layer ($K = 3 \times 10^{-6}$) and bottom layer ($K = 10^{-6}$).**

**Run an animation and examine the ‘activeness’ of local and regional flow systems.**

**What are some implications on GW-SW interactions?**

## Effects of anisotropic $K$

Remove the middle and bottom layers.
Specify $K_x = 10^{-5}$ and $K_z = 2 \times 10^{-6}$ m s$^{-1}$.



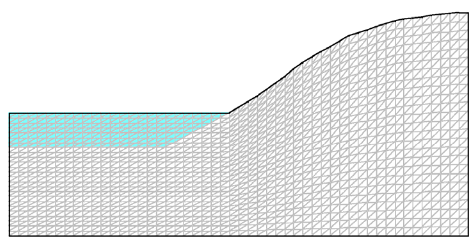
Flow lines are oblique to hydraulic head contours.
Flow paths are shallower compare to the isotropic case. Why?



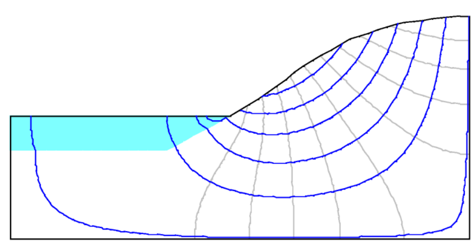
## GW-Lake interaction

We can represent a lake in TopoDrive by a zone with very high $K$.

Why?



Set up a cross section with a lake represented by $K$ = 0.01 m s$^{-1}$.



Flow lines are denser near the shore, indicating higher discharge flux along the shore.

While this explains general spatial trends of lake-bottom seepage flux, in reality, the flux distribution is much more complex due to geological heterogeneity.
→ Explore this further by yourself.