Baseflow Analysis

Objectives

1. Understand the conceptual basis of baseflow analysis.
2. Estimate the baseflow component of stream hydrographs.

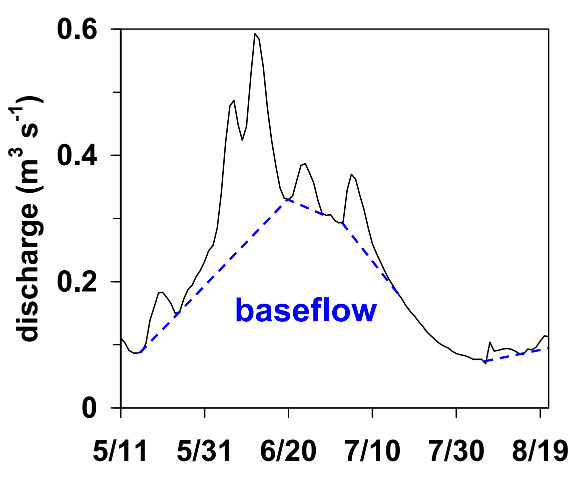
Baseflow definition and significance

Portion of (stream) flow that comes from groundwater or other delayed sources (Tallaksen, 1995. *J. Hydrol.*, 165: 349).

Understanding of low-flow condition is important for water resource management and environmental protection.

→ Why?

We will review:
(1) Baseflow recession analysis
(2) Baseflow separation technique

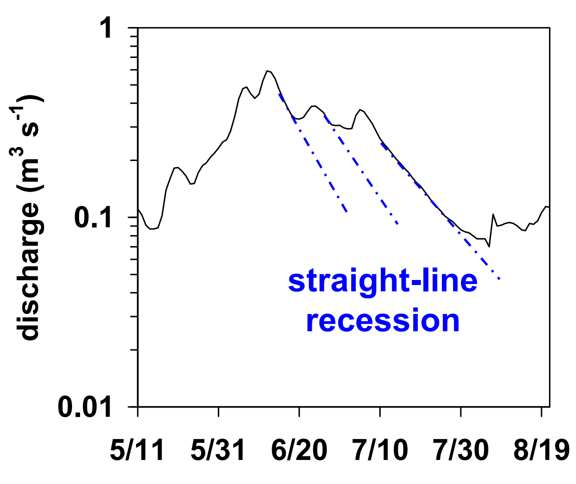


Stream discharge gradually decreases after storm events.

Various baseflow 'separation' techniques have been proposed.

What purpose?

Regardless of sophisticated algorithms, they are all arbitrary.



Recession hydrographs commonly plot as straight lines on a semi-log graph.

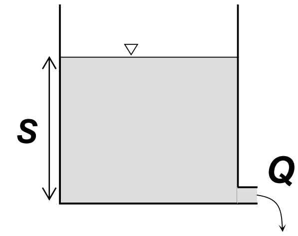
$Q(t) = Q_0\exp(-at)$

Q_0 : discharge at $t = 0$

a : constant (s^{-1})

What causes the exponential behaviour?

Reservoir model for recession analysis



Exponential function is the solution of:

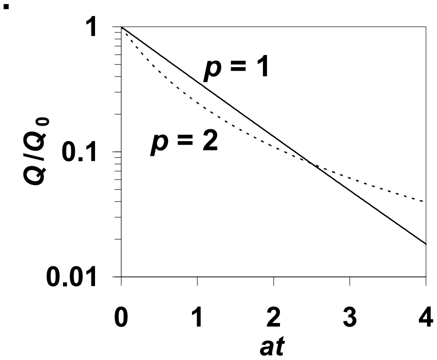
$$Q = aS \quad \text{and} \quad \frac{dS}{dt} = -Q \qquad \text{(linear reservoir)}$$

S: volume of water stored (m^3)

A more general reservoir model is given by:

$$Q = aS^p \quad \text{and} \quad \frac{dS}{dt} = -Q$$

p: dimensionless constant



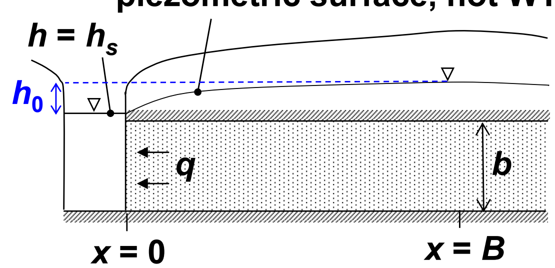
Non-linear (p > 1) reservoir represents the effects of complex processes such as the transmissivity feedback.

The solution of the non-linear reservoir equation is:

$$Q(t) = Q_0(1 + at)^{-p/(p-1)}$$

See Tallaksen (1995) for a comprehensive review.

Physically-based aquifer model



Baseflow from a homogeneous, confined aquifer is described by:

$$\frac{\partial}{\partial x}\left(Kb\frac{\partial h}{\partial x}\right) = S_s b \frac{\partial h}{\partial t}$$

$$T\frac{\partial^2 h}{\partial x^2} = S_c \frac{\partial h}{\partial t}$$

$T = Kb$: transmissivity ($m^2 s^{-1}$)

$S_c = S_s b$: storage coefficient or storativity

Boundary conditions and the initial condition

$h(x_0) = h_s$ for all $t > 0$

No flow at the divide ($x = B$) for all $t > 0$

$h = h_s + h_0$ at $t = 0$ for all $0 \le x \le B$

Rorabaugh (1964. *Int. Assoc. Scientific Hydrol. Pub.* 63: 432-441, Eq.1) reported the Fourier-series solution for the flow per shore length, q ($m^2 s^{-1}$):

$$q = (2Th_0/B)[\exp(-at) + \exp(-9at) + \exp(-25at) + \ldots]$$

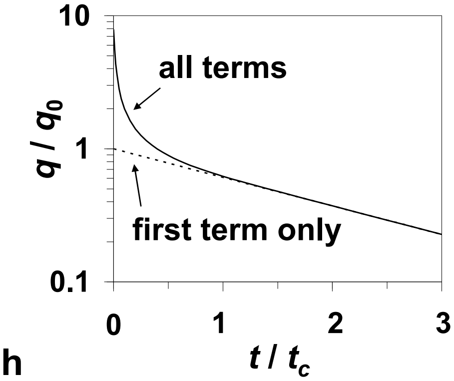
where $a = \pi^2 T / (4B^2 S_c)$

High-order terms in the series are negligible for $t / (B^2S_c/T) > 0.2$.

$t_c = 0.2\,B^2S_c / T$ is called 'critical time'.

$\therefore q \cong q_0 \times \exp\{-at\}$ for $t > t_c$

where $q_0 = 2Th_0/B$



Note the similarity between this and the exponential decay equation of hydrograph in Page 2.

→ What does this mean?

Remember the recession coefficient in this model: $a = \pi^2 T / (4B^2S_c)$.

The recession coefficient in Page 2 represents the average properties of aquifer over the entire watershed.

Note: $T / S_c = K / S_s = D_h$ ← Hydraulic diffusivity (unit?)

What is the unit of a? Unit of $1/a$?

→ Hydrologic response time

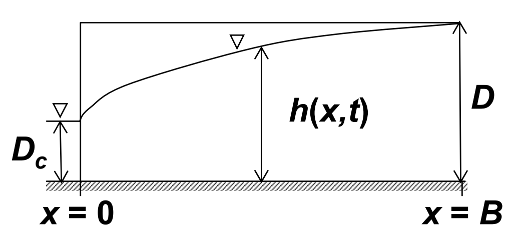
Models for unconfined aquifer

Streams are usually connected to unconfined, not confined, aquifers. Rigorous analysis of unconfined aquifers would require solutions of the Richards equation.

The Dupuit-Forchheimer (D-F) approach offers a reasonable approximation of complex problems (e.g., Paniconi et al., 2003. *Water Resour. Res.*, 39: 1317). The transient flow equation based on the D-F approximation is called the Boussinesq equation:

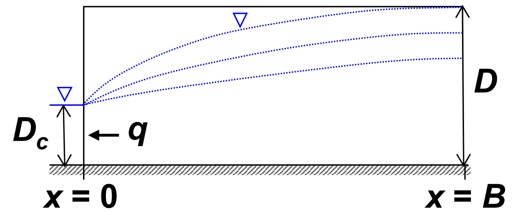
$$\frac{\partial}{\partial x}\left(Kh\frac{\partial h}{\partial x}\right) = S_y\frac{\partial h}{\partial t}$$

K: aquifer conductivity (m s^{-1})
S_y: drainable porosity



Exact solutions of the non-linear Boussinesq equation is available only for special cases. Brutsaert (2005, *Hydrology – an introduction*. Ch. 10, Cambridge Univ. Press) presented a summary of various solutions for the cross section shown above.

Drainage of riparian aquifer



A riparian aquifer becomes fully saturated after a heavy storm (t = 0).

This solution considers gradual drainage of a hillslope after some time.

q: flow per shore length ($m^2 s^{-1}$)

Approximate analytical solution is obtained by 'linearizing' the Boussinesq equation:

$$\frac{\partial}{\partial x}\left(Kh_m \frac{\partial h}{\partial x}\right) = S_y \frac{\partial h}{\partial t} \longrightarrow Kh_m \frac{\partial^2 h}{\partial x^2} = S_y \frac{\partial h}{\partial t}$$

where h_m is the 'average' saturated thickness.

Solving the linearized equation, drainage flux per shoreline is:

$$q = (2Kh_m D/B)[\exp(-at) + \exp(-9at) + \exp(-25at) + \dots]$$

where $a = \pi^2 K h_m / (4B^2 S_y)$

This is almost identical to the Rorabaugh (1964) equation.

Therefore, for $t > 0.2B^2 S_y / Kh_m$

$$q \cong (2Kh_m D/B) \exp[-\pi^2 K h_m t / (4B^2 S_y)]$$

Brutsaert (2005, p.400) proposed expressing h_m as a fraction of D:

$h_m = pD$ where $p \cong 0.35$ for Dc << D
$p \cong (D + D_c)/(2D)$ for other cases Eq. [2]

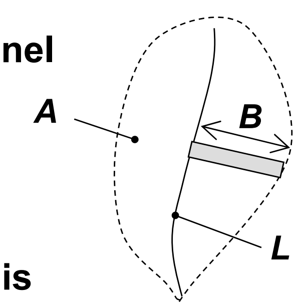
Using p, the flux is written as (Brutseart, 2005, Eq.10.116):

$$q = (2KpD^2/B) \exp\{-\pi^2 KpDt / (4B^2 S_y)\}$$

We note that the average distance from the channel to drainage divide, B, can be estimated by:

$B = A / (2L)$

→ Why?



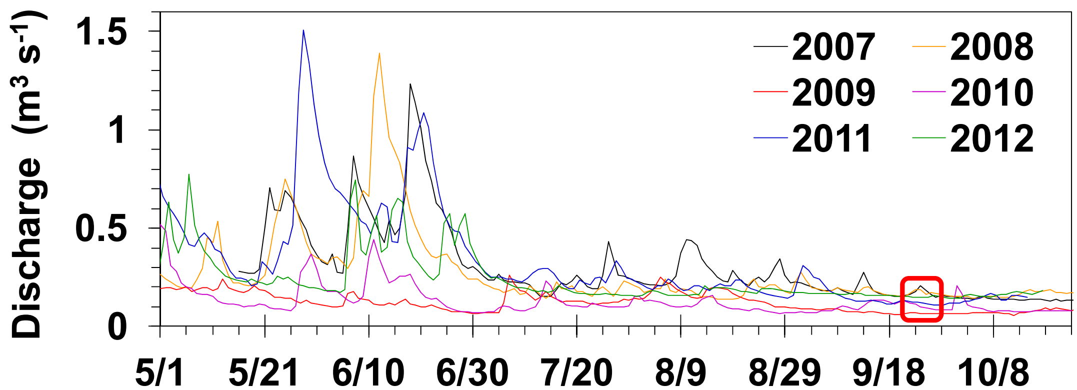
Total baseflow Q ($m^3 s^{-1}$) at the watershed outlet is (Brutseart, 2005, Eq.10.164):

$$Q = 2L \times (2KpD^2 2L/A) \exp\{-\pi^2 KpDt\, 4L^2/(4A^2 S_y)\}$$
$$= (8KpD^2 L^2/A) \exp\{-\pi^2 KpDL^2 t/(A^2 S_y)\} \quad \text{Eq. [3]}$$

Watershed-scale behavior of baseflow is exponential.

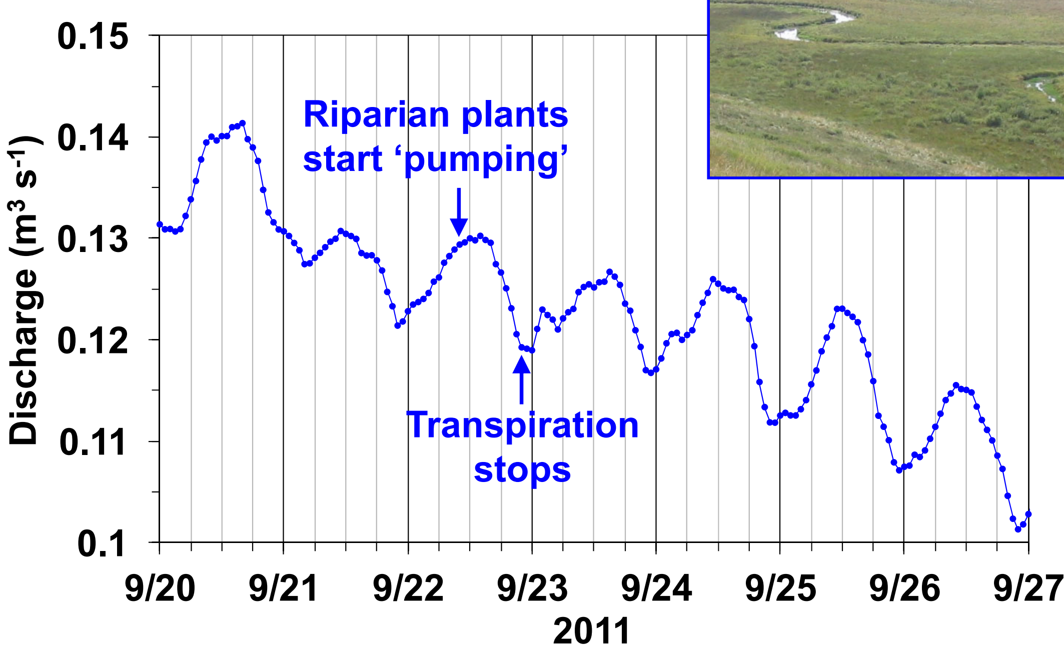
The decay coefficient contains information on hydraulic properties of the watershed.

Real Stream Example: West Nose Creek, Calgary

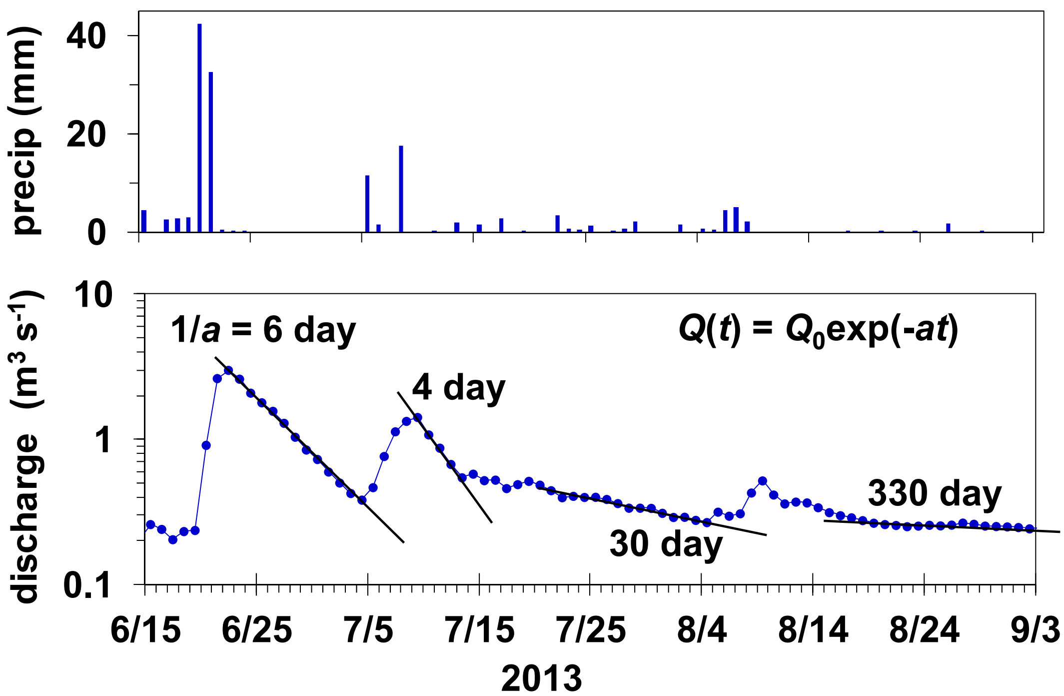


Hayashi and Farrow (2014. *Hydrogeol. J.* 22: 1825-1839)

Riparian SW-GW Interaction



Exponential Decay?

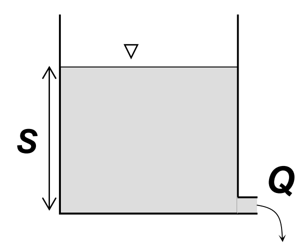


Why does the decay coefficient vary?

Linear reservoir and exponential recession

$Q = Q_0\exp(-at)$ is the solution of:

$$Q = aS \quad \text{and} \quad \frac{dS}{dt} = -Q \qquad \text{(linear reservoir)}$$



In reality,

$Q = f(S)$ ← Complex function (non-linear, hysteretic)

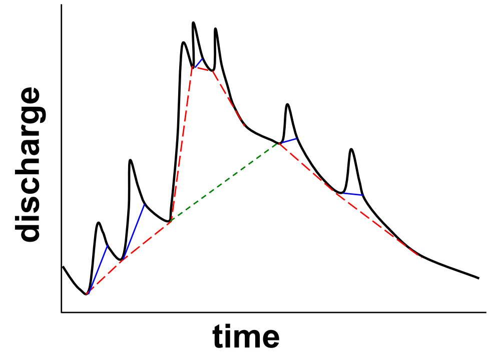
$$\frac{dS}{dt} = -Q - E$$ ← Evapotranspiration

The catchment scale storage-discharge function, $f(S)$ still contains useful information. → See Kirchner (2009, *Water Resour. Res.* 45, W02429).

Baseflow separation

Given a hydrograph, 'quick' flow and baseflow can be separated by a number of different methods.

- **Connecting local minima**
- **Variation of local-minima method**
- **Using inflection points**



All methods use arbitrary criteria for baseflow, and are time consuming for manual operation.

Automated techniques are at least objective, and are efficient for processing many data sets.

We will use a digital-filter algorithm of Arnold et al. (1995. *Ground Water*, 33: 1010) **to demonstrate the usefulness and limitation of automated baseflow separation.**

Recursive digital filter

The algorithm, originally described by Nathan & McMahon (1990. *Water Resour. Res.* 26: 1465), **calculates the quick flow component q_i at time step *i* from q_{i-1} at previous time step and total flow Q_i and Q_{i-1}:**

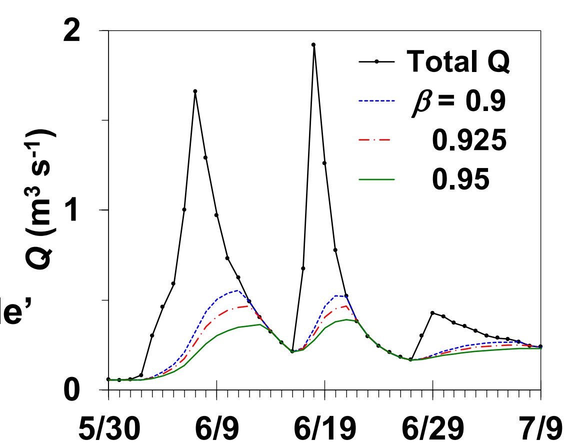
$$q_i = \beta q_{i-1} + \frac{1+\beta}{2}(Q_i - Q_{i-1})$$

where β is a filter constant ranging between 0.9 and 0.95.

Baseflow b_i is calculated as: $b_i = Q_i - q_i$

In this example from the Marmot Creek watershed in 2005, the filter was applied with three different values of β.

The case with β = 0.95 appears to have produced the most 'reasonable' separation result.



Baseflow index

By applying the digital filter to the entire 2009 summer discharge data set (May 1- September 10) for Marmot Creek, it was found that:

Total discharge = 2.48×10^6 m^3
Total baseflow = 1.77×10^6 m^3

The ratio of total baseflow to discharge is base flow index (BFI).

In this example, BFI = 1.77 / 2.48 = 0.76.

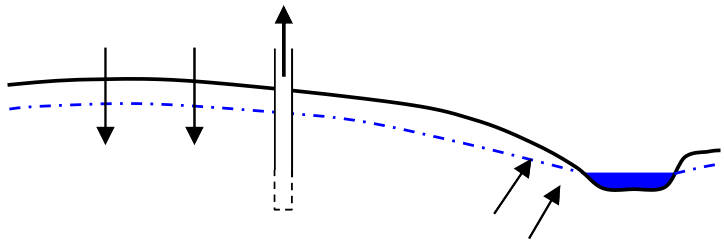
Automated baseflow separation offers a convenient tool to calculate BFI for multiple watersheds having different size and geology, or for a single watershed in multiple years having different meteorological forcing or land-use practice.

We will use a computer program Baseflow with a sample data set from the Marmot Creek watershed in a computer exercise to calculate BFI.

Computer exercise: Baseflow separation

Groundwater is connected to streams

Water balance is key to understand the connection



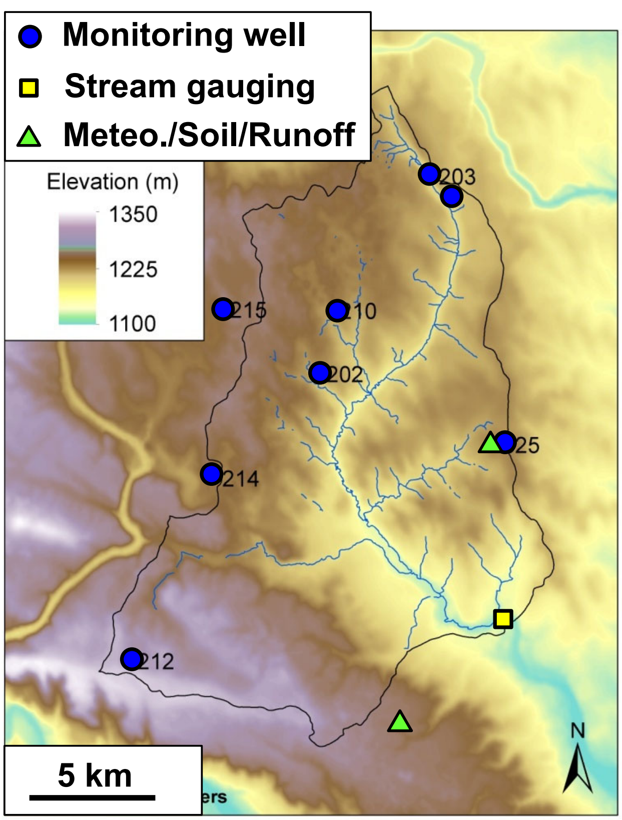
Recharge − Discharge − Pumping = Storage Change
(water level ↑↓)

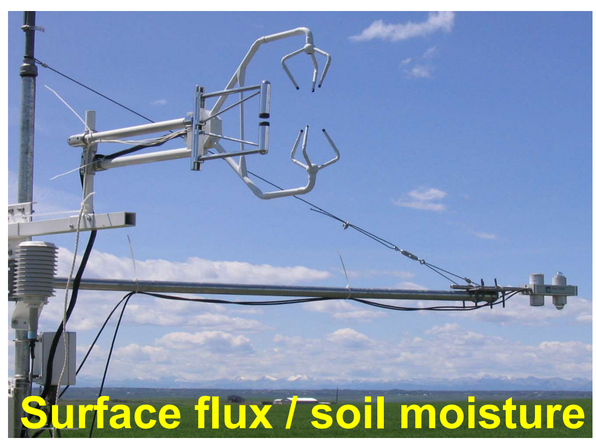
Long-term balance (storage change ≈ 0):

Recharge ≈ Discharge + Pumping

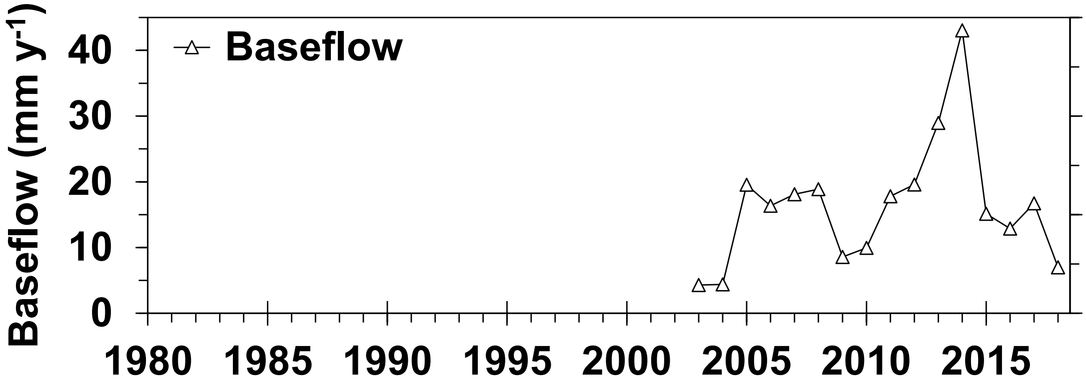
Use baseflow to estimate watershed-scale recharge.

West Nose Creek Hydrological Observatory





West Nose Creek Baseflow



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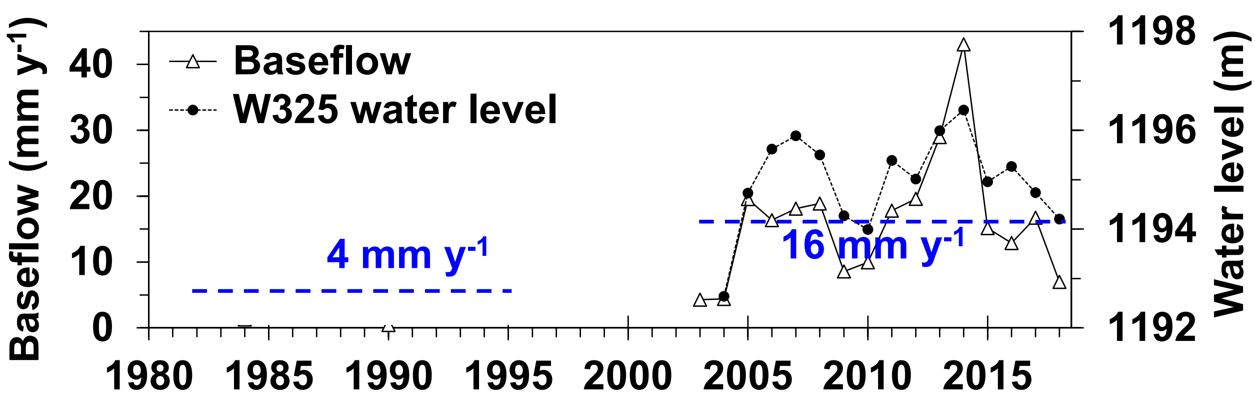
West Nose Creek Baseflow



Recharge ≈ Discharge + Pumping

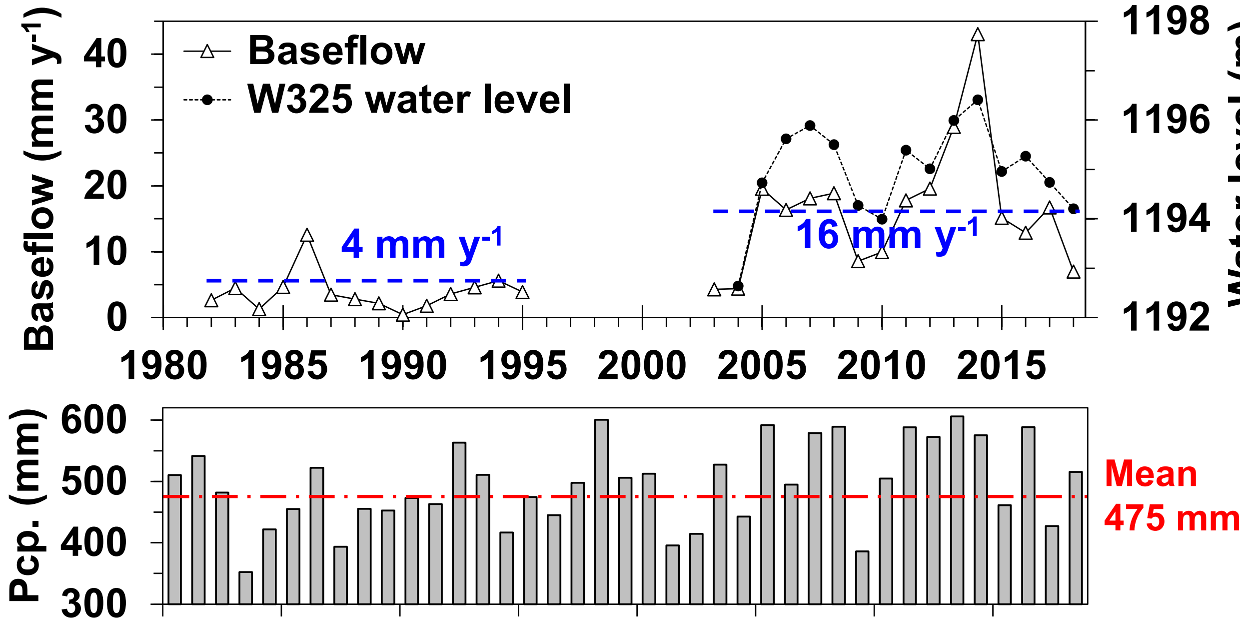
Total groundwater extraction ≈ 2-3 mm y^{-1}

Recharge ≈ 6-7 mm y^{-1} in 1982-1995

18-19 mm y^{-1} in 2003-2018

Hayashi and Farrow (2014. *Hydrogeol. J.* 22: 1825-1839)

West Nose Creek Baseflow



Present recharge ≈ 18-19 mm y^{-1}, much larger than GW extraction rate of 3 mm y^{-1}.

→ What if the drier condition of the 1980s returns?

Hayashi and Farrow (2014. *Hydrogeol. J.* 22: 1825-1839)