Groundwater Hydrology

Instructor: Masaki Hayashi, University of Calgary, Canada

Motivation, relevance, importance?

Essential resource for life

Dodoma, Tanzania

Large-scale water supply

Kumamoto, Japan

Headwater of streams

Canadian Rockies

Source of dry-season river flow

Pokhara, Nepal

Importance of dry-season flow?

- Aquatic environment
- Water supply
- Irrigation
- Run-of-river hydropower generation
-

Groundwater affects pore pressure in sediments.

→ Reduce the shear strength. → Trigger mass movement.

Landslide

Earthflow

Highland & Bobrowsky (2008, The landslide handbook. U.S. Geol. Survey Circular 1325)

Course Objectives and Outline

Learning objectives is to understand:

- Fundamental concepts of groundwater flow and storage
- Roles of groundwater in the hydrologic cycle
- Groundwater resource management

Outline

1. Groundwater flow and storage processes
2. Aquifer and aquitard
3. Measurement of groundwater
4. Groundwater flow system
5. Vadose zone processes
6. Surface water – groundwater interaction

Textbook: Hornberger et al. (2014, Elements of physical hydrology, p.145-168, 173-193) → H2014

Course Logistics

Computer exercise

TopoDrive program – Windows PC only
Baseflow separation – Microsoft Excel

Calculation problem set

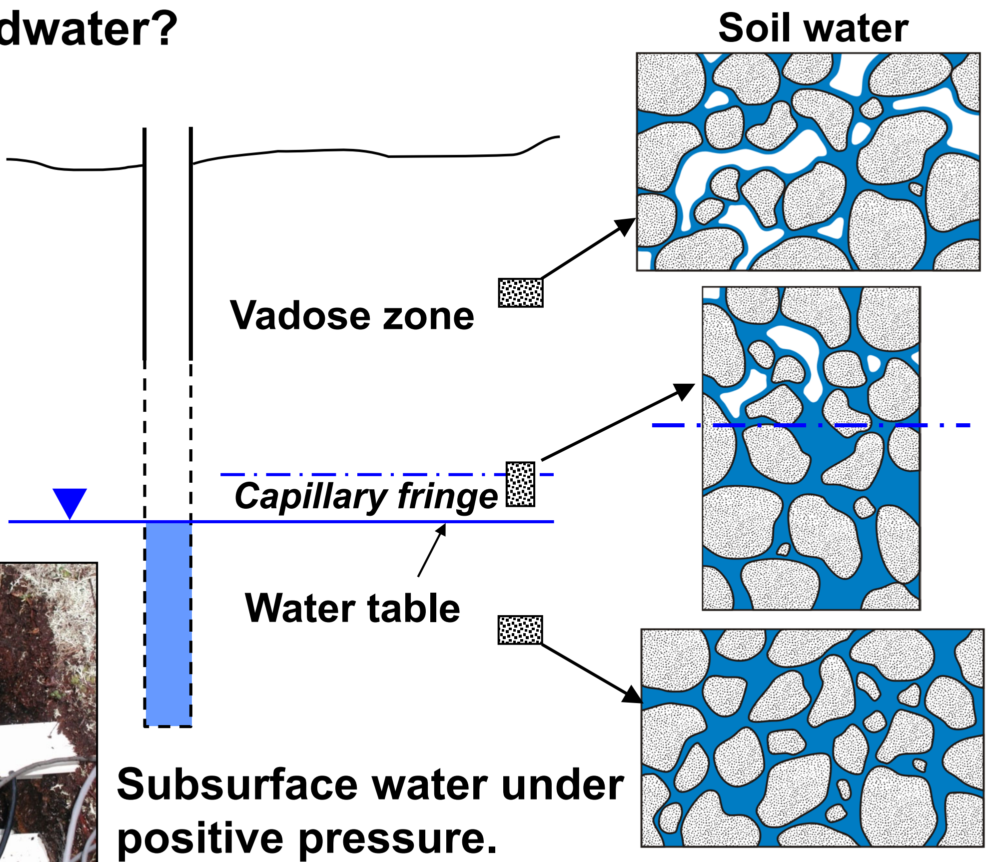
Solved during lectures.

Report

Short report on computer exercises and problem set.
Report preparation time on Wednesday afternoon.
Report is due on June 20, 9:00 (submitted on Moodle).

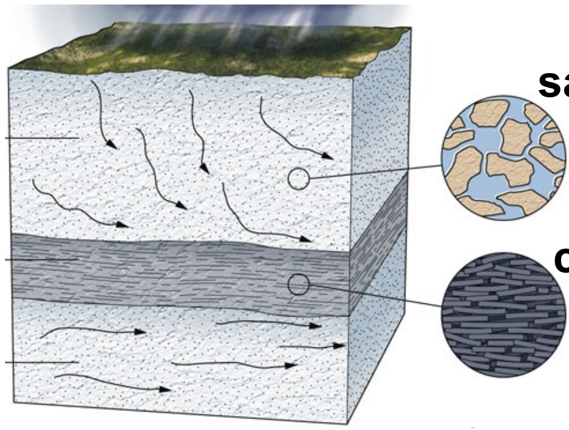
Lec. 1: Fundamentals of Groundwater Hydrology

What is groundwater?



Where do we find groundwater?

- Pore spaces in unconsolidated sediments
- Thin fractures in rocks



Marshak (2001. *Earth: Portrait of a planet*)

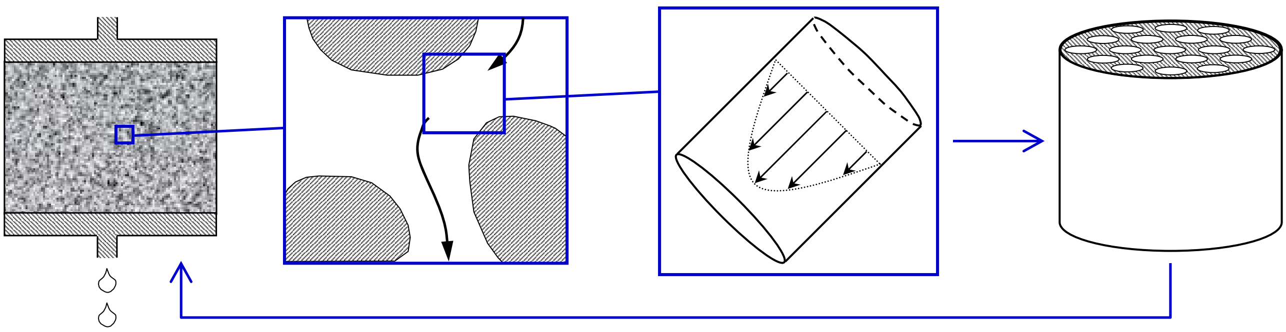
Fractured sandstone

<u>Essential features of groundwater flow</u>

1. Small pore diameter (typically 10^{-6} to 10^{-3} m)
2. Balance between driving forces and viscous 'friction'
3. Assemblage of numerous flow pathways

Conceptual Model of Groundwater Flow

- Mathematical abstraction of essential features
  → Syukuro Manabe's 'motto' in early years of his career.
- Scientific reasoning supported by observation



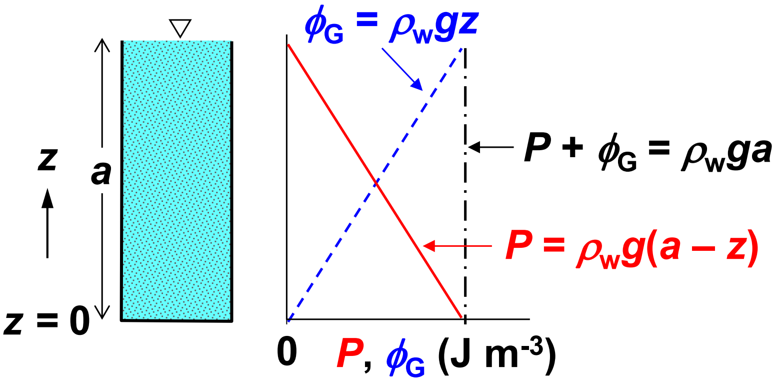
Force balance: Driving force ↔ Resistance

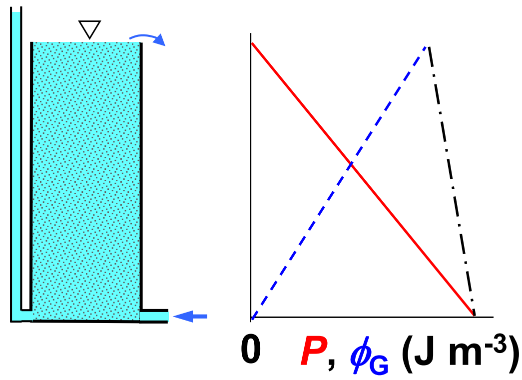
pressure / gravity — friction ← viscosity

Pressure and Gravitational Potential (GP)

GP of a solid object = mgz (J) m: mass (kg)
z: elevation (m)

GP of a unit volume of water = $\rho_w gz$ (J m^{-3}) $\leftarrow \phi_G$





Hydraulic head (h): $h = \dfrac{P + \phi_G}{\rho_W g}$

h is constant $\rightarrow \dfrac{dh}{dz} = 0$

Hydrostatic condition.

h decreases with z $\rightarrow \dfrac{dh}{dz} < 0$

Upward flow.

More about hydraulic head:

$$h = \frac{P + \phi_G}{\rho_W g} = \frac{P}{\rho_W g} + \frac{\phi_G}{\rho_W g} = \frac{P}{\rho_W g} + \frac{\rho_W gz}{\rho_W g} = \psi + z$$

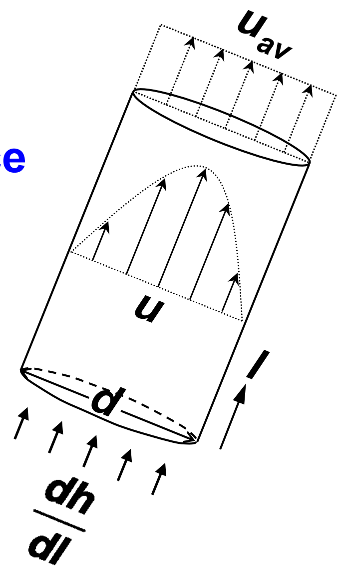
ψ: Pressure head (m)
$\rightarrow$ Hydrostatic pressure at the measurement point.

Poiseuille's equation

Mathematical model of flow through a thin tube having a diameter *d* (m).

- Hydraulic head gradient = $\frac{dh}{dl}$ ← driving force
- Flow resistance by viscosity μ (kg m^{-1} s^{-1}).
- Flow velocity (u, m s^{-1}) has a parabolic distribution.
- Average velocity (u_{av}) = $-\frac{d^2}{32}\frac{\rho_w g}{\mu}\frac{dh}{dl}$

→ H2014, p.149



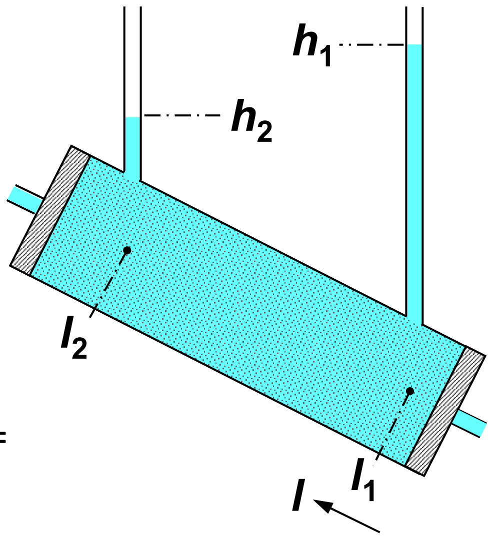
Poiseuille's equation provides a physics-based building block of fluid flow through small pores and thin fractures.

How small?

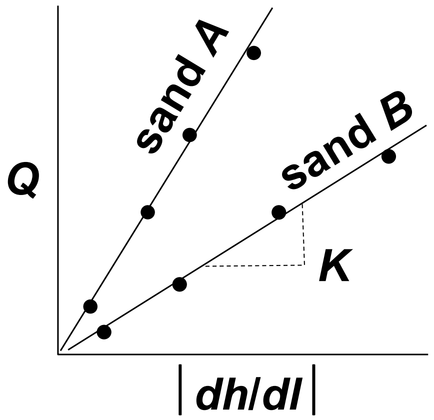
Darcy's Law

Flow through a pipe packed with saturated sand.

Q: flow rate (m^3 s^{-1})
A: cross-sectional area (m^2)
l: distance along the tube (m)



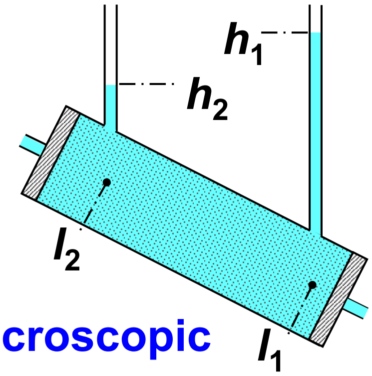
$$\frac{dh}{dl} =$$



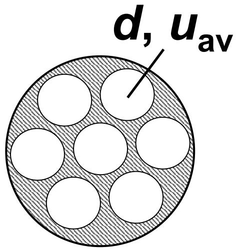
$$\frac{Q}{A} = q = -K\frac{dh}{dl} = -K\frac{h_2 - h_1}{l_2 - l_1}$$

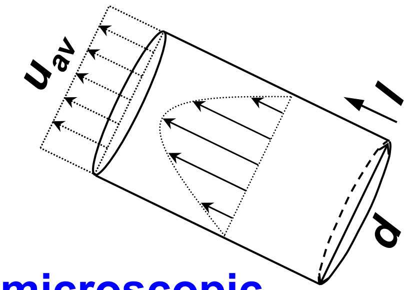
q: specific discharge (m s^{-1})
= flow per unit area
K: hydraulic conductivity (m s^{-1})

Darcy's law and Poiseuille's equation



macroscopic





microscopic

$$q = -\boxed{K}\frac{dh}{dl} \quad \longleftrightarrow \quad u_{av} = -\boxed{\frac{d^2}{32}\frac{\rho_w g}{\mu}}\frac{dh}{dl}$$

$$K = k\frac{\rho_w g}{\mu}$$ k: intrinsic permeability (m^2)

Permeability is related to pore size.

$$k \cong C d_m^{\ 2}$$

C: empirical coefficient
d_m: representative pore diameter (m)

- Darcy's law applies to water, oil, gas, etc.
  → Density and viscosity will vary.

$$K = k\frac{\rho_w g}{\mu}$$

- Dependence of k on $d_m^{\ 2}$:
  10^2 folds increase in d_m → 10^4 folds increase in k and K.

- ρ_w = 10^3 kg m^{-3}, g = 9.8 m s^{-2}, $\mu \cong 10^{-3}$ kg m^{-1} s^{-1} at 20 ^{o}C.
  → $\rho_w g / \mu \cong 10^7$ m^{-1} s^{-1}.

Example: Estimating K of sediments

Assume C = 10^{-3}; d_m = 0.3 mm for sand, 0.01 mm for silt.

k = for sand, for silt

K = for sand, for silt

Sediment Particle Size Distribution

Sediment particles have a wide range of diameters (*d*).
They are divided into four size classes:

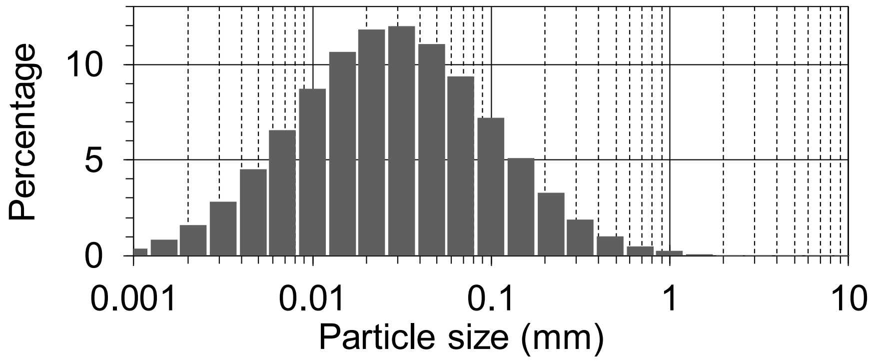
- **Clay: *d* < 0.002 mm**
- **Silt: 0.002 < *d* < 0.063 mm (= 1/16 mm)**
- **Sand: 0.063 < *d* < 2 mm**
- **Gravel: *d* > 2 mm**

US National Highway Inst. (2006)

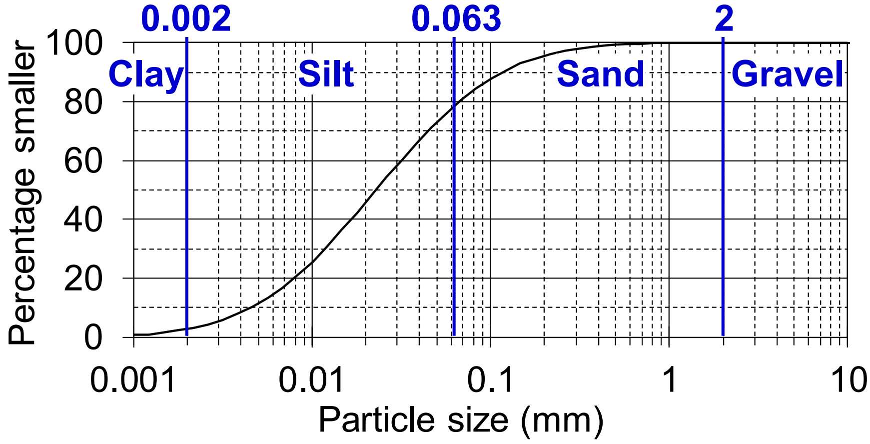
Analytical procedure:

1. **Sample is dried and put through mechanical sieves to analyze particles with *d* > 2 mm.**
2. **Finer particles are chemically treated and analyzed using a laser particle analyzer or a hydrometer.**
3. **Size distribution is reported by weight percentage.**

Frequency of occurrence for *d* < 2 mm portion of the sample.



Same data are plotted as cumulative frequency distribution.

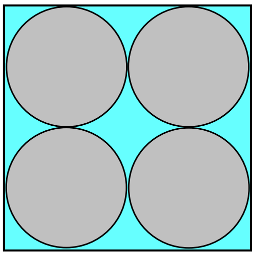


Clay % =

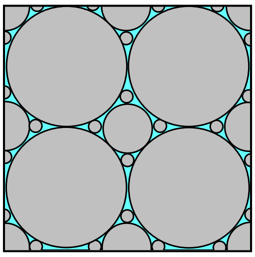
Silt % =

Sand % =

Representative pore diameter

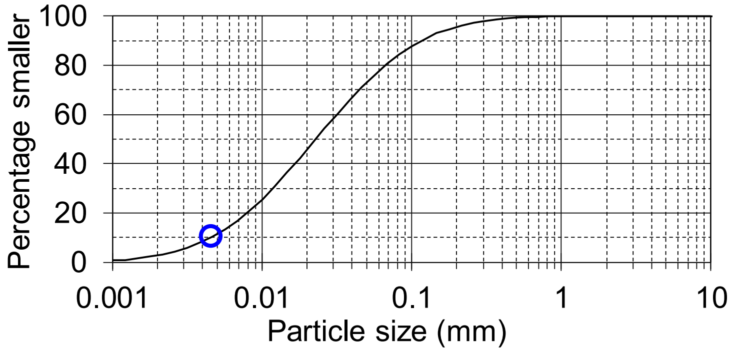


For uniform material with a single grain size, the pore diameter is similar to the grain diameter.

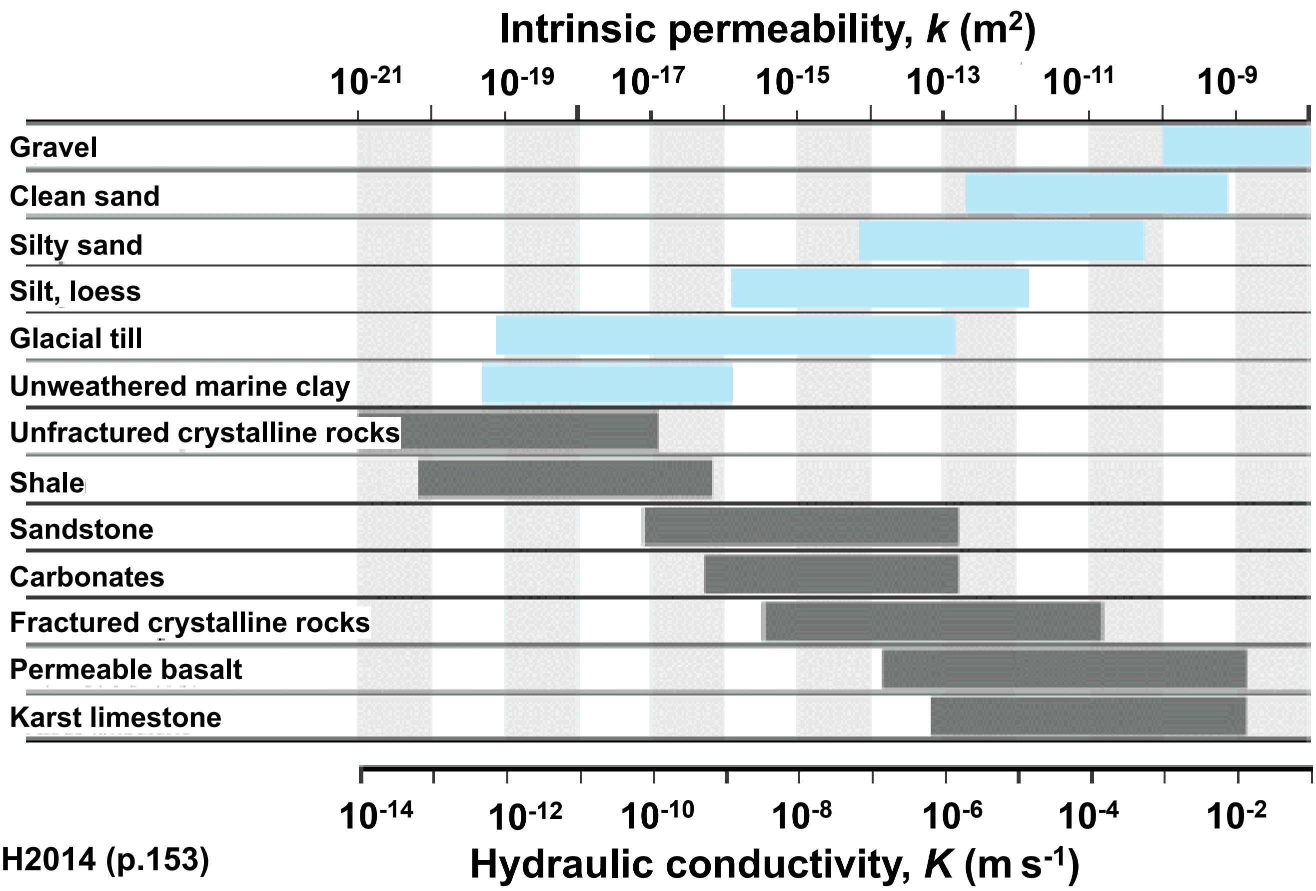


For non-uniform material, the pore diameter is similar to the diameter of the smallest grains.

From the grain size distribution, d_{10} is commonly used as d_m.



Ranges of *K* and *k* of geological materials



H2014 (p.153)

Porosity = Void Volume / Total Volume

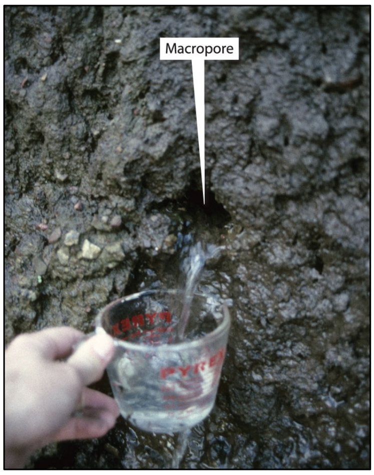
Primary porosity

Material	Porosity
Clay	0.4-0.7
Silt	0.35-0.5
Sand	0.25-0.5
Gravel	0.25-0.4
Shale	0.0-0.1
Sandstone	0.05-0.3
Limestone	0.0-0.2
Crystalline rock	0.0-0.05

Freeze & Cherry (1979. *Groundwater*)

Secondary porosity

Fractured sandstone

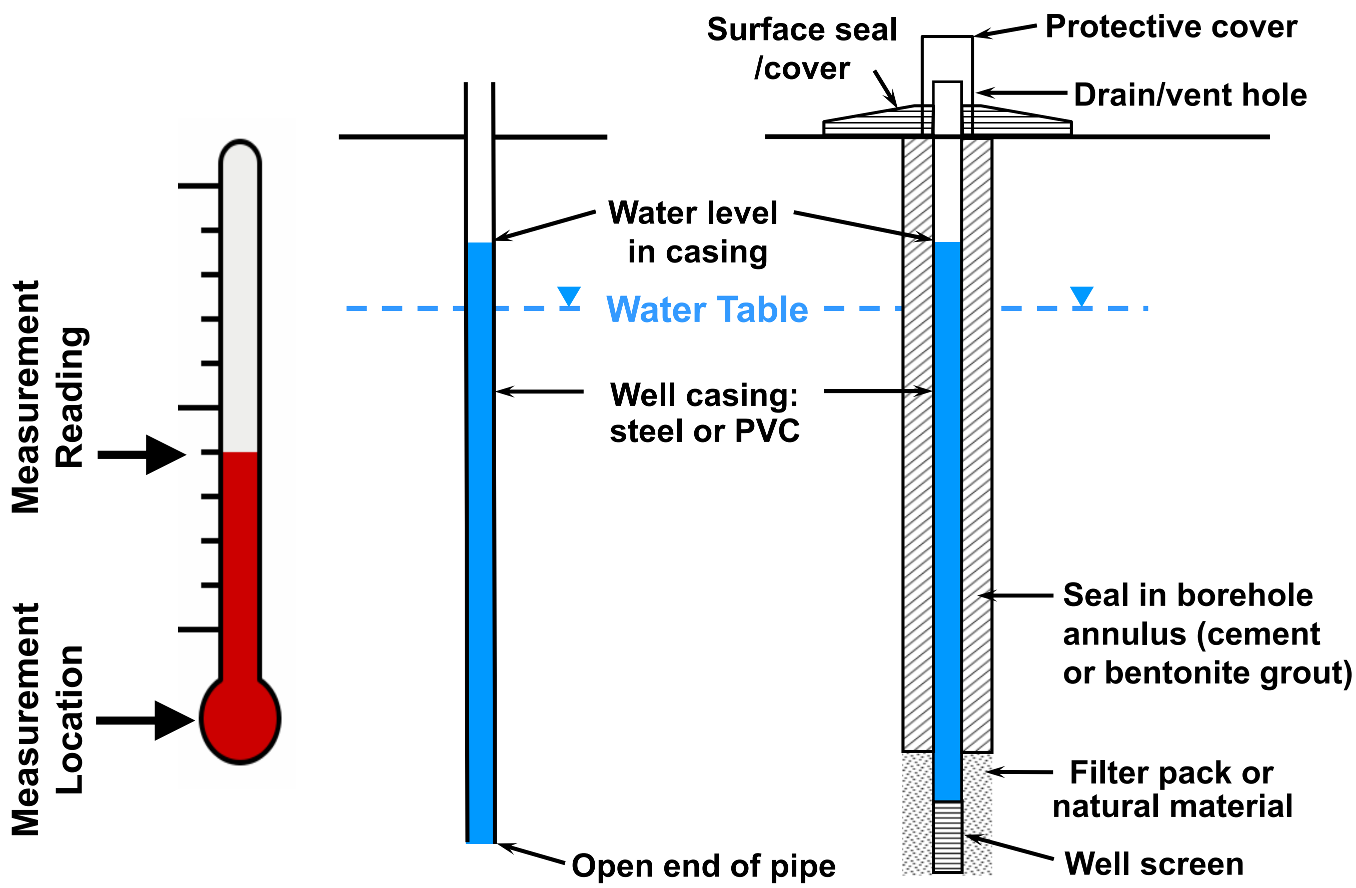


Soil macropore

Bierman & Montgomery (2020)

These features can increase the permeability by orders of magnitude.

Measurement of Hydraulic Head: Piezometer



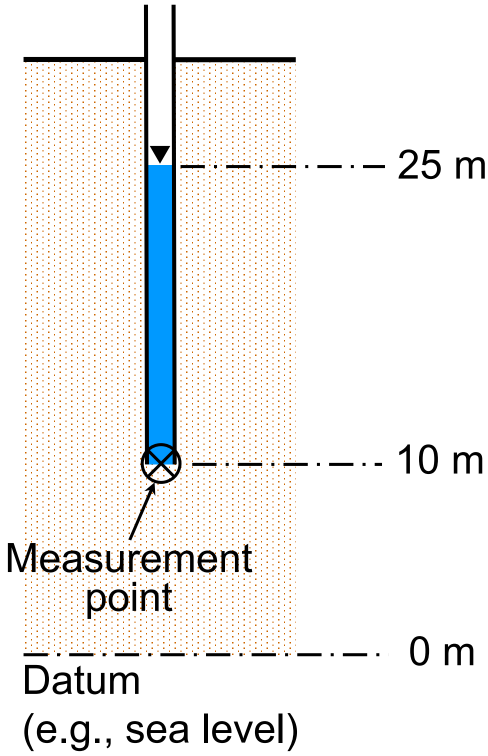
What do water levels mean in practical terms?

Consider the water level measured in a piezometer.

h = hydraulic head at a point is the elevation of the top of the static water column above the point (relative to the datum)

z = elevation of the point above the selected datum

ψ = height of the water column above the point



<u>Example</u>:

h = ***z*** =

ψ =

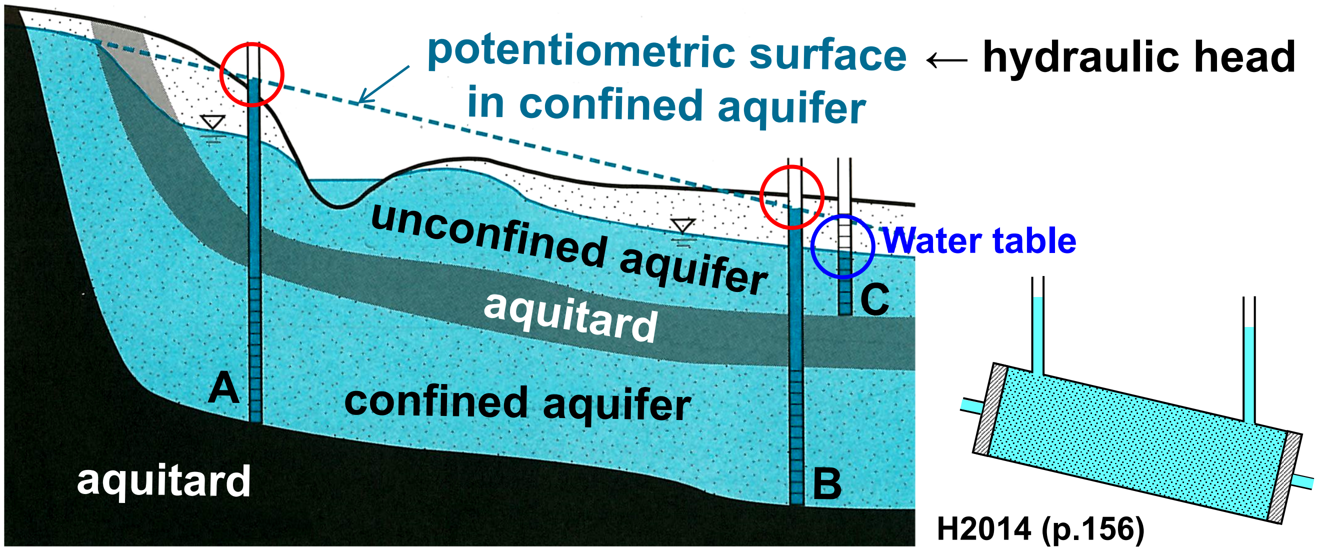
<u>Examples of piezometer installation</u>

Photos by Edwin Cey

Aquifers and Aquitards

Aquifer : Saturated permeable geologic unit that can transmit significant quantities of water
→ groundwater 'reservoir', e.g., sand, gravel, fractured rock

Aquitard : Saturated geologic unit that is incapable of *transmitting* significant quantities of water.
→ confining layer, e.g., clay, shale, non-porous bedrock



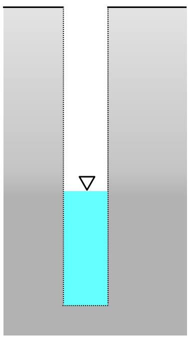
H2014 (p.156)

Confined aquifer : Confined at the top and the bottom by aquitards. Also called artesian aquifer.

Some confined aquifers have potentiometric surface above the ground (see well A in the last slide).
→ flowing condition.

Unconfined aquifer : The water table defines the upper surface. Unconfined aquifers are usually shallow and vulnerable to groundwater contamination.

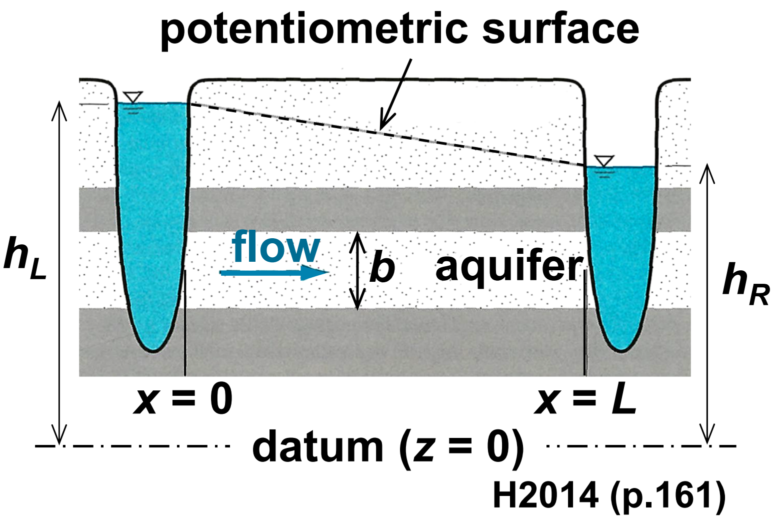
Water table: Surface of zero water pressure (P = 0).



Darcy's Law in a Confined Aquifer

$$q = -K\frac{dh}{dx} = -K\frac{h_R - h_L}{L}$$

$$= K\frac{h_L - h_R}{L}$$



Assume:

$K = 10^{-4}$ m s^{-1}

h_L = 190 m, h_R = 186 m

L = 4 km

q =

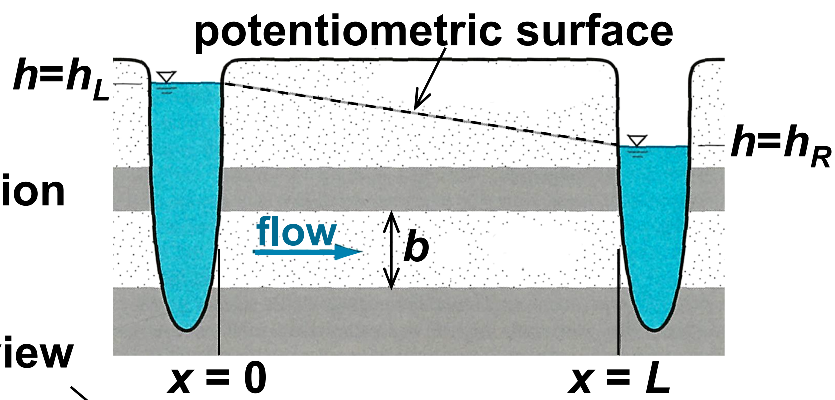
Suppose aquifer width is w (m). Total flow rate Q (m^3 s^{-1})?

$Q = qA = qbw$

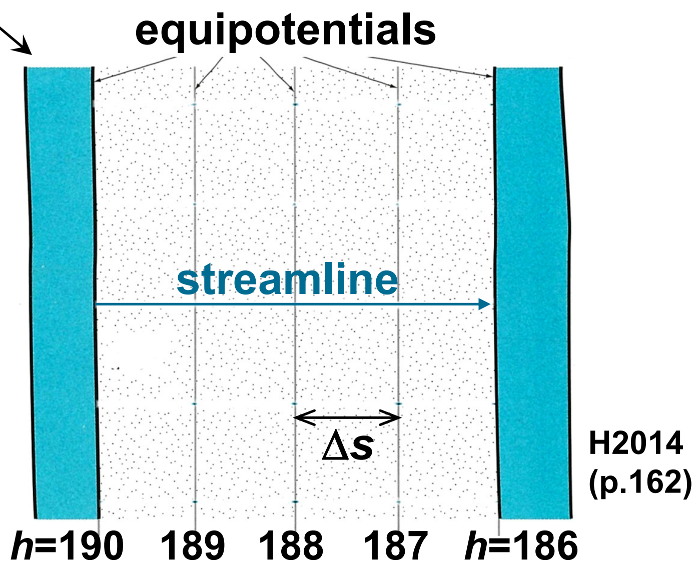
Assume b = 10 m, w = 200 m.

Q =

Vertical cross section



Plan view



Contour lines of constant hydraulic head (i.e. equal potential) → <u>equipotentials</u>

e.g., Δh = 1 m

Distance between lines (Δs) depends on h_L and h_R.
→ <u>boundary conditions</u>

Flow direction (<u>streamline</u>) is normal to equipotentials.

Draw more streamlines with a spacing Δm so that:

$\boxed{\Delta m \cong \Delta s}$ ← important!

This is called <u>flownet</u>.

Discharge from a 'flow tube':

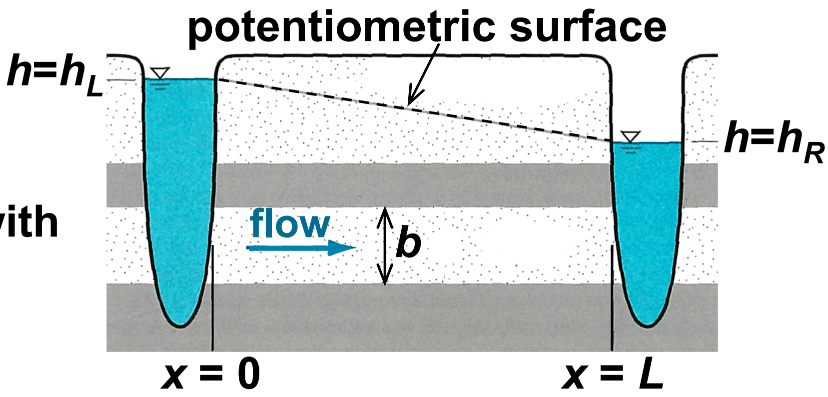
$$Q = qA = q \times b\Delta m$$

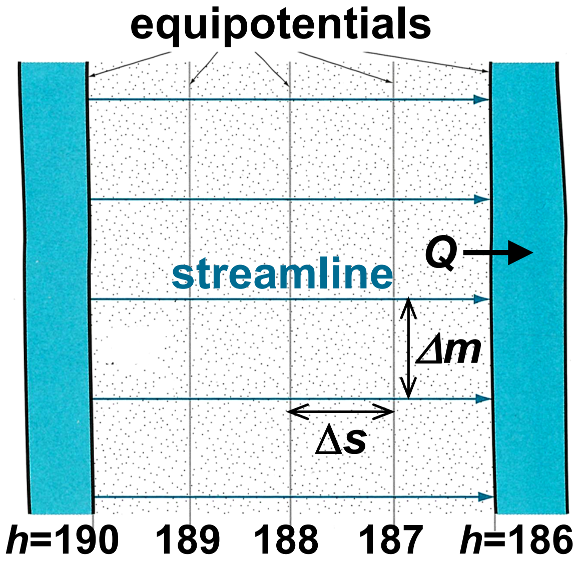
$$= K\frac{\Delta h}{\Delta s} b\Delta m = Kb\Delta h\frac{\cancel{\Delta m}}{\cancel{\Delta s}}$$

$$\therefore\ Q = Kb\Delta h$$

$K = 10^{-4}$ m s^{-1}, $b = 10$ m

Total flow between rivers = ?



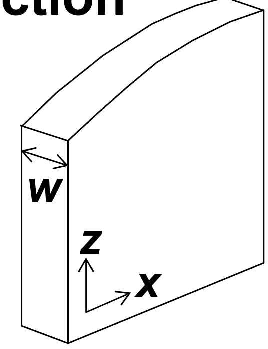


H2014 (p.165)

Flownet for Idealized Hill-Valley Cross Section

The Hubbert (1940) Section

Equipotentials and streamlines are drawn so that $\Delta m \cong \Delta s$ for each 'curvilinear' squares.



Suppose:

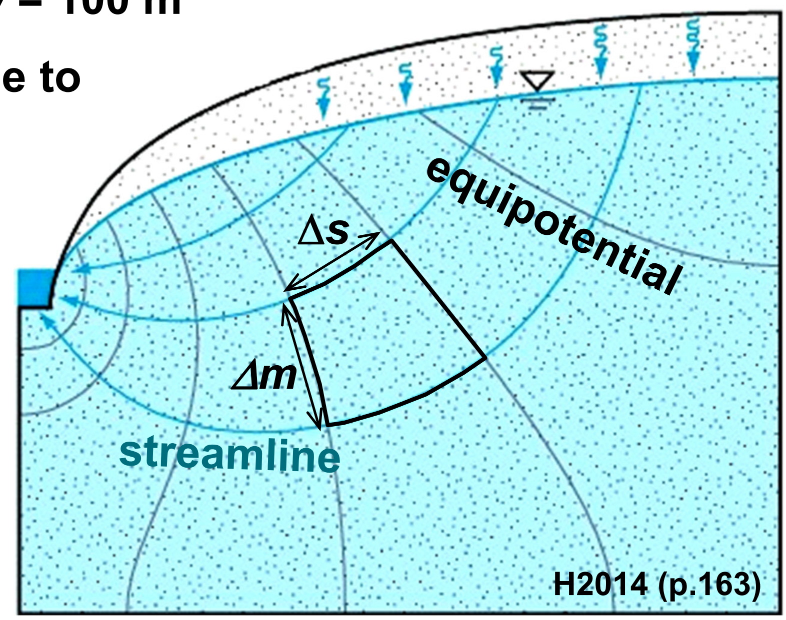
$\Delta h = 1$ m, $K = 10^{-6}$ m s^{-1}, $w = 100$ m

Discharge from the hillslope to stream in this reach?

For each flow tube,

$Q = Kw\Delta h\ =$

Density of flow tubes?

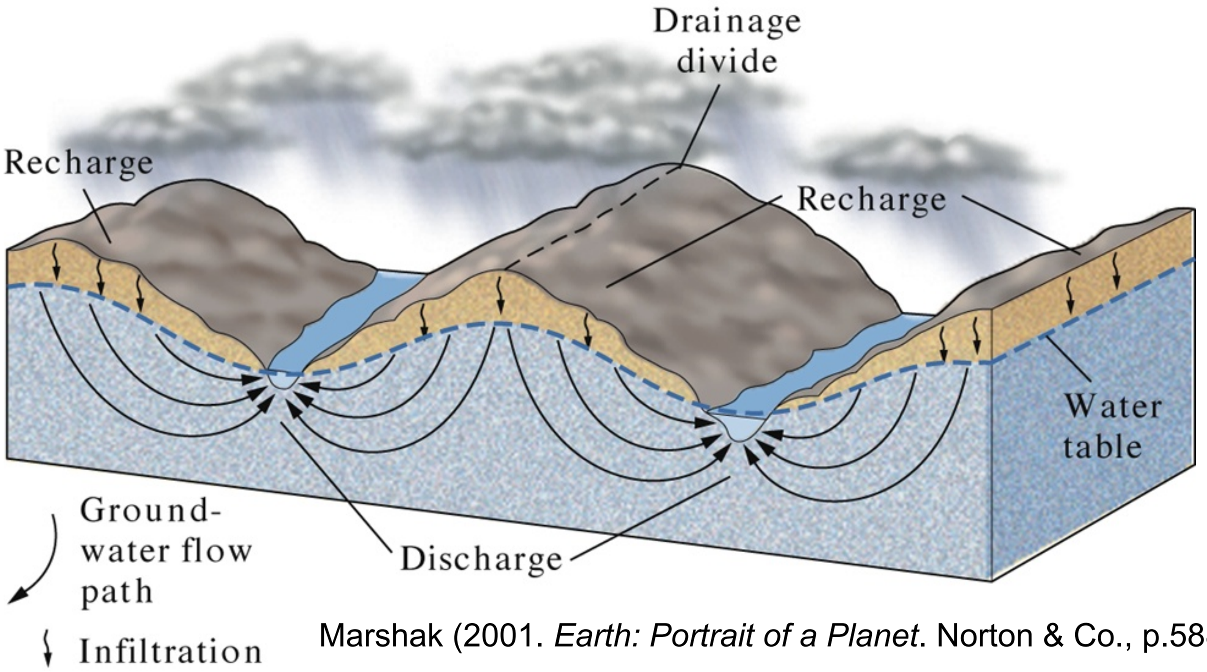


H2014 (p.163)

Groundwater Flow System

Two fundamental principles in groundwater hydrology:

1. **Water table is a subdued replica of ground surface.**
2. **Groundwater flows from high to low.**



Marshak (2001. *Earth: Portrait of a Planet*. Norton & Co., p.588)

Darcy's Law in Three Dimensions

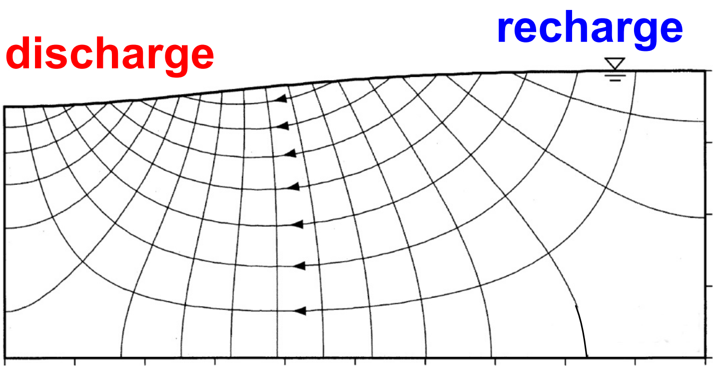
$$q_x = -K_x \frac{\partial h}{\partial x} \qquad q_y = -K_y \frac{\partial h}{\partial y} \qquad q_z = -K_z \frac{\partial h}{\partial z}$$

q_x, q_y, q_z : specific discharge (m s^{-1})
K_x, K_y, K_z : saturated hydraulic conductivity (m s^{-1})
h : hydraulic head (m)

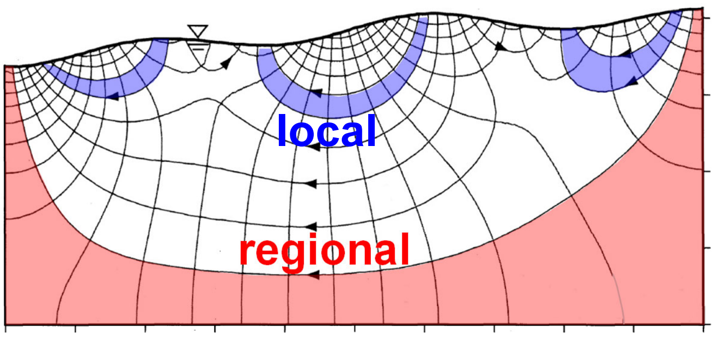
Topo Drive Exercise

Effects of Topography

The water table mimics the surface topography
→ subdued replica of land surface.



H2014, p.181



H2014, p.183

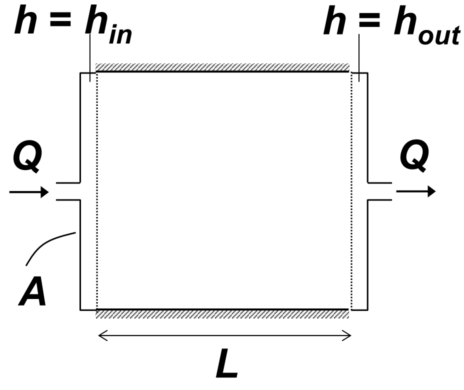
Add local topography.

'Nested' GW flow system

- **Shallow local system**
- **Deep regional system**

Geological Heterogeneity and Anisotropy

Water is flowing through a box at Q ($m^3\ s^{-1}$). We are not given the information on the material. We use Darcy's law to assign the 'bulk' hydraulic conductivity (K_b) of the box.



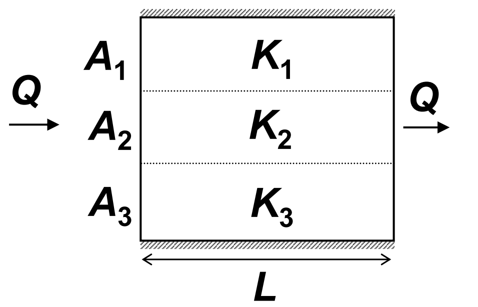
$$Q = Aq = -AK_b \frac{h_{out} - h_{in}}{L} = -K_b \frac{A\Delta h}{L}$$

$$\therefore K_b = -\frac{QL}{A\Delta h}$$

K_b is dependent on the property of sediments and how they are arranged in the box. Let's examine two important cases.

(1) Parallel layers

$$Q = -A_1K_1\frac{\Delta h}{L} - A_2K_2\frac{\Delta h}{L} - A_3K_3\frac{\Delta h}{L}$$



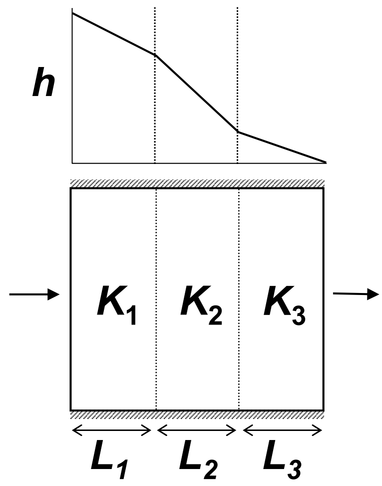
$$\therefore K_b = \frac{K_1A_1}{A} + \frac{K_2A_2}{A} + \frac{K_3A_3}{A} \quad \text{Eq. (1.1)}$$

K_b is the <u>weighted arithmetic average</u> of each material. The layer thickness serves as a <u>weighting factor</u>. Thicker layers have a heavier influence on K_b than thinner layers.

(2) Serial layers

In this case, *q* needs to be the same in all layers.

$$-K_b\frac{\Delta h}{L}=q=-K_1\frac{\Delta h_1}{L_1}=-K_2\frac{\Delta h_2}{L_2}=-K_3\frac{\Delta h_3}{L_3}$$



Also, the head drop in each layer must add up to the total head drop Δh.

$$\Delta h_1+\Delta h_2+\Delta h_3=\Delta h$$

$$\therefore K_b=\left(\frac{L_1}{K_1L}+\frac{L_2}{K_2L}+\frac{L_3}{K_3L}\right)^{-1} \quad \text{Eq. (1.2)}$$

K_b is given by the <u>weighted harmonic average</u> of each material.

Parallel layers: $K_b=\frac{K_1A_1}{A}+\frac{K_2A_2}{A}+\frac{K_3A_3}{A}$ **Eq. (1.3)**

Serial layers: $K_b=\left(\frac{L_1}{K_1L}+\frac{L_2}{K_2L}+\frac{L_3}{K_3L}\right)^{-1}$ **Eq. (1.4)**

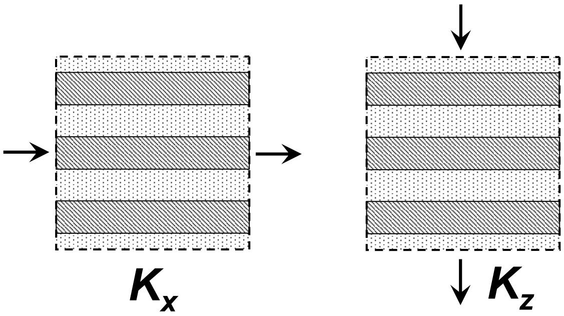
<u>Example</u>

Suppose

$A_1 = A_2 = A_3 = A/3$ $L_1 = L_2 = L_3 = L/3$ $K_1 = K_3$, $K_2 = 0.01\,K_1$

What is the K_b for the parallel and serial case?

Suppose a layered sediment: sand and clay

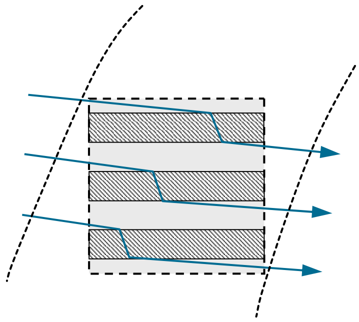


$K_x >> K_z$

K **is dependent on direction:**
<u>anisotropic</u> material.

Note: Each layer consists of <u>isotropic</u> material.
Small scale heterogeneity → large scale anisotropy.

Streamlines may not be normal to equipotentials in anisotropic materials.



Topo Drive Exercise

(anisotropy)

Aquifer Transmissivity

For horizontal flow in a confined aquifer, <u>transmissivity</u> (T, m^2 s^{-1}) is defined by K_h and the thickness of the aquifer:

$$T = \sum_i K_{hi} d_i = \int_0^D K_h(z)dz \qquad \text{Eq. (1.5)}$$

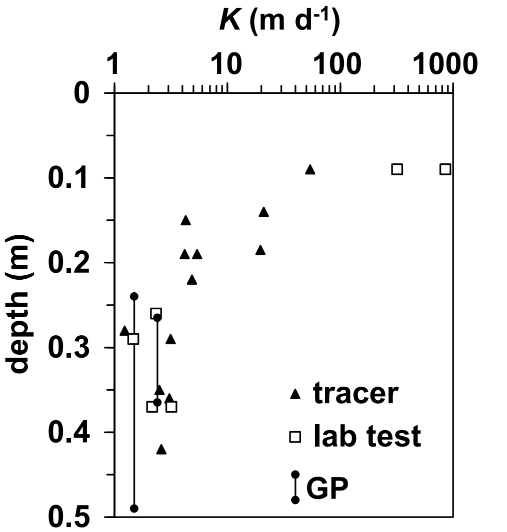
K_{hi}: horizontal conductivity (m s^{-1}) of individual layers
d_i: thickness (m) of individual layer
D: total aquifer thickness (m)

For the resource evaluation of confined aquifers, constant K_h is usually assumed. In this case, $T = K_h \times D$.

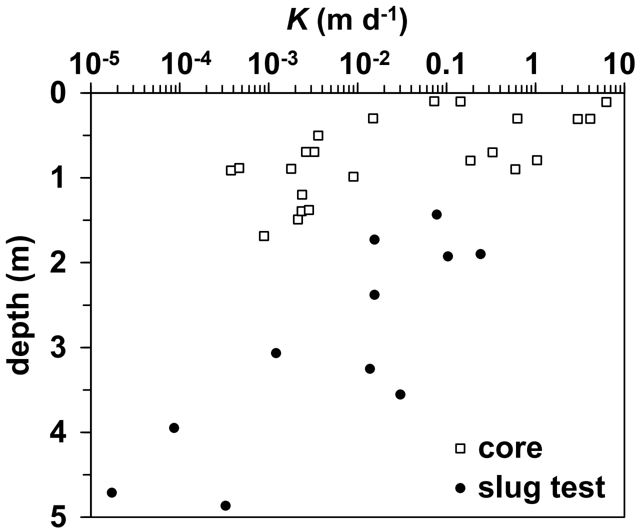
Transmissivity is also defined for unconfined aquifers using Eq. (1.5). However, D is variable in unconfined aquifers, because their top boundary is defined by the water table.

In general, K in shallow (< 5 m) sediments decreases with depth. Below are examples of organic soils from the subarctic region (left) and glacial sediments from the prairie region (right) of Canada, measured by various methods.

K decreases dramatically with depth.

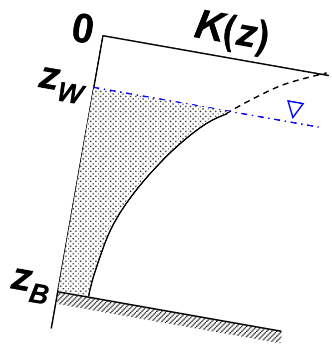


Quinton et al. (2008. *Hydrol. Process.*, 22:2829). GP: Guelph permeameter



Parsons et al. (2004. *Hydrol. Process.*, 18:2011)

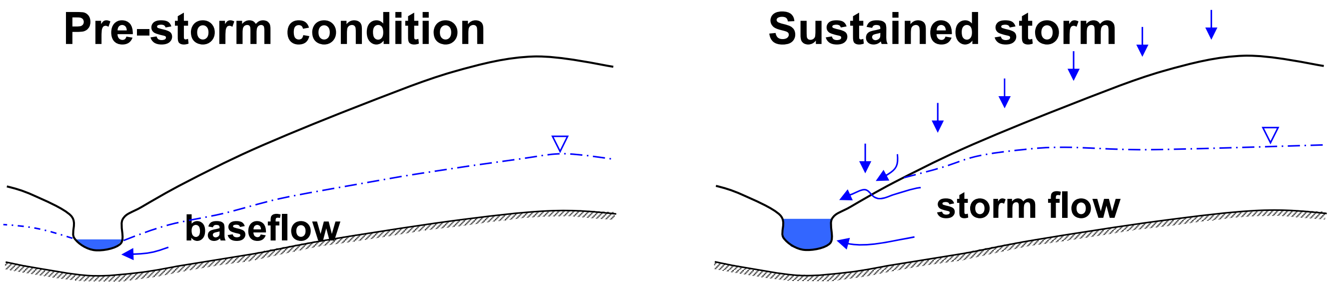
The depth dependence of $K(z)$ is particularly important for groundwater flow through hillslope sediments underlain by bedrock. The hillslope transmissivity is:



$$T(z_W) = \int_{z_B}^{z_W} K(z)dz$$

K is integrated from a relatively impermeable bedrock surface (z_B) to the water table (z_W).

Transmissivity feedback



The rising water table along a stream channel reaches the ground surface → high T → rapid groundwater flow.

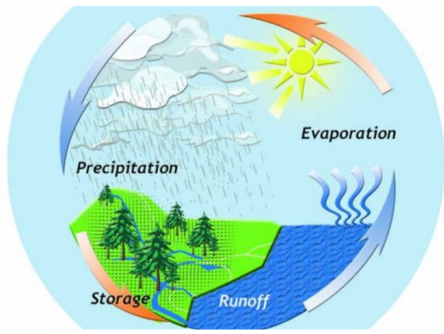
Groundwater Contribution to Storm Flow

Stable isotope tracer of water (H_2O)

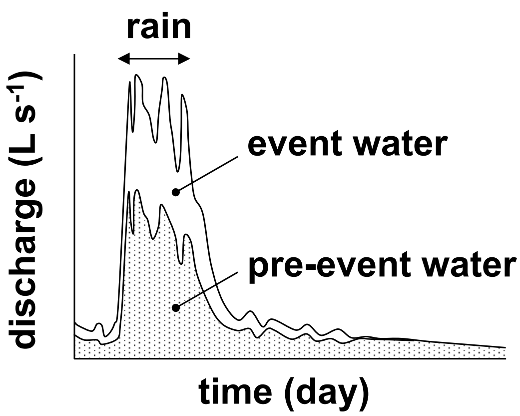
H has an atomic weight of 1, O has 16.
→ 1+1+16 = 18 for 'normal' water.

Some O has a weight of 18 → ^{18}O isotope.

Isotope concentration is a natural tracer of water through the hydrologic cycle.



http://www.gewex.org



Modified from Buttle (1994. *Progress in Physical Geography*, 18:26)

Using stable isotopes (^2H, ^{18}O), the 'event' water resulting from a storm can be distinguished from the pre-event water released from storage.

Pre-event water contributes typically 50-75 % of storm flow in low-order watersheds (e.g., Buttle, 1994).

Storage of Water in Unconfined Aquifer

How are storage changes (ΔV) in aquifers related to changes in hydraulic head (Δh)?

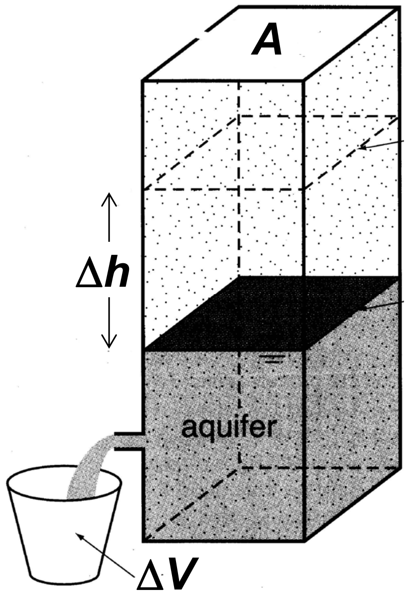
Consider a change in volume per unit area of an aquifer.

For an <u>unconfined aquifer</u>:

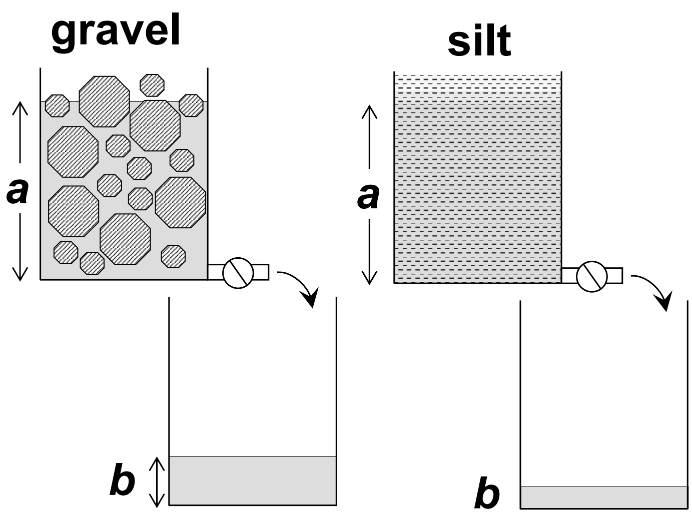
$$\frac{\Delta V}{A} = S_y \Delta h$$

S_y: specific yield
unit?

How is S_y related to the types of aquifer material?



H2014, p.188



$$\frac{b}{a} = \frac{\text{drainage per area}}{\text{water table drop}}$$

→ Specific yield (S_y)

For gavel and coarse sands
$S_y \cong$ porosity

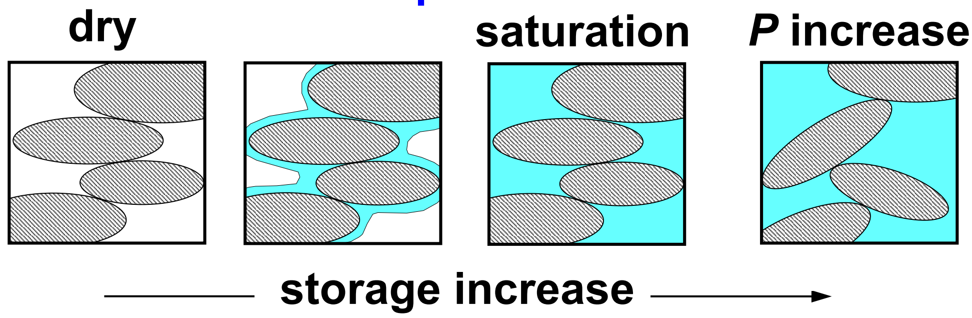
For finer size material
S_y << porosity

Above definition of S_y assumes that:
(1) Draining or filling of pores is instantaneous.
(2) Ratio b/a is independent of the depth to the water table.

Are these assumptions valid?

Storage of Water in Confined Aquifer

unconfined aquifer



Grains move.
→ expansion of pore spaces

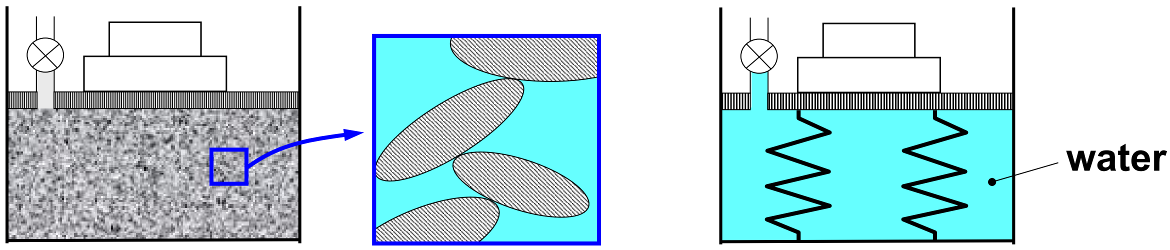
Storage change mechanisms in confined aquifers:

Primary: Elastic expansion/compression of pores.
- Compressibility of sediments

Secondary: Change in density of water with pressure.
- Compressibility of water

<u>Compressibility of sediments</u>

Consider saturated sand in a rigid container with a movable top plate and a weight.



The saturated sand can be viewed as a water-spring system. Spring represents the solid skeleton of sand grains.

Suppose we inject a small volume of water while keeping the constant weight. What will happen to the top plate?
→ The top plate will rise slightly, and the springs stretch.

This implies that the force supported by the spring will decrease. But the weight remains constant. What will happen to the pressure of water? → Increase.

Many experiments have shown that the rise of the top plate is proportional to the amount of pressure increase.

Will the volume of sand grains change with water pressure?

→ No. The volume change is due to an increase in porosity.

Compressibility of sediments α (Pa^{-1}) is defined by

$$\alpha = \frac{1}{V}\frac{dV_{void}}{dP} \cong \frac{dn}{dP}$$

V: bulk volume (m^3) of the sediments
V_{void}: void volume (m^3) in the sediments
n: porosity

<u>Compressibility of water</u>

Water is usually regarded as incompressible fluid. However, a small amount of compression may have significant effects in groundwater storage. Why?

Compressibility (β, Pa^{-1}) of water is defined as:

$$\beta = -\frac{1}{V_w}\frac{dV_w}{dP}$$

Volume decreases as pressure increases.

V_w: volume (m^3) of water in the box
P: pore water pressure (Pa)

Using ρ_w = density (kg m^{-3}) of water = $\frac{\text{mass}}{V_W}$

$$\beta = \frac{1}{\rho_w}\frac{d\rho_w}{dP}$$

Density increases as pressure increases.

Putting the two mechanisms together

From the consideration of compressibility of sediments and water, the change in water volume (ΔV_w) in a volume (V) of bulk sediment is given by:

$$\Delta V_w = VS_s\Delta h \qquad S_s = \rho_w g(\alpha + n\beta)$$

***g*: gravitational acceleration ($\cong$ 9.8 m s^{-2})**

S_s is called specific storage, and can be understood as the 'volume of water extracted from a unit volume of sediments by a 1-m drop of pressure head'.

For most sediments, S_s ranges from 10^{-5} to 10^{-2} m^{-1}.

Examples

Using the table below, estimate:

S_s of plastic clay with n = 0.50

S_s of fractured sandstone with n = 0.05

Summary of material storage parameters

Compressibility (α)

Material	α (Pa^{-1})
Plastic clay	$2 \times 10^{-6} - 3 \times 10^{-7}$
Stiff clay	$3 \times 10^{-7} - 1 \times 10^{-7}$
Loose sand	$1 \times 10^{-7} - 5 \times 10^{-8}$
Dense sand	$2 \times 10^{-8} - 1 \times 10^{-8}$
Sandy gravel	$1 \times 10^{-8} - 5 \times 10^{-9}$
Rock, fissured	$7 \times 10^{-10} - 3 \times 10^{-10}$
Rock, sound	$< 3 \times 10^{-10}$
Water (β)	4.8×10^{-10}

Porosity (n)

Material	n (%)
Gravel	24 – 38
Coarse sand	31 – 46
Fine sand	26 – 53
Silt	34 – 61
Clay	34 – 60
Sandstone	5 – 30
Siltstone	21 – 41
Shale	0 – 10
Crystaline rocks	0 – 5
Fractured crystaline rocks	0 – 10

Domenico and Schwartz (1998. *Physical and Chemical Hydrogeology*, 2nd ed. Wiley, pp.15 & 66).

Storativity of confined aquifer

Volume change per unit area in confined aquifer is:

$$\frac{\Delta V}{A} = S_s b \Delta h$$

b: aquifer thickness (m)

$S_s b$ is called aquifer storativity (S). Unit of S?

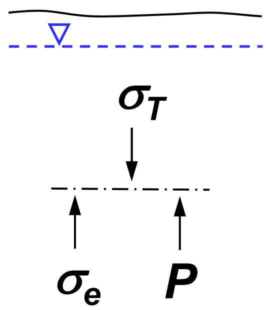
Aquifer transmissivity (see slide 39) and storativity depend on:
- Material property (K and S_s)
- Aquifer thickness (b)

Transmissivity and storativity are important variables for groundwater resource evaluation.

→ How much water can we extract from an aquifer?

More on Compression and Expansion

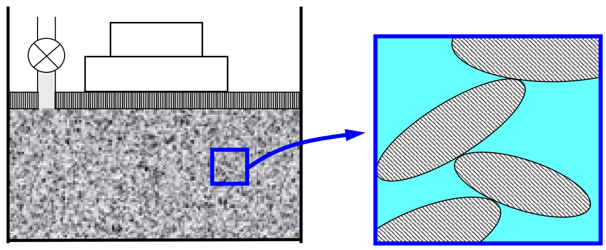
Effective stress

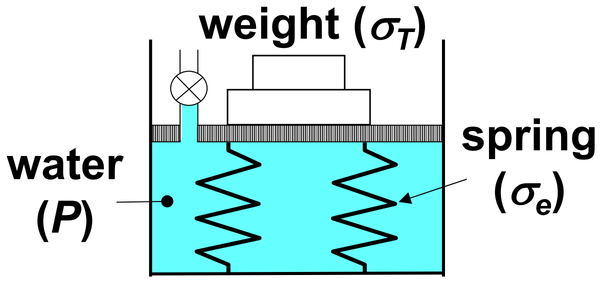


An imaginary plane at some depth is pushed by the weight of water and sediments above. This force per unit area is total stress (σ_T, Pa).

The weight is supported by solid 'skeleton' of sediments and water. The force transmitted by solid skeleton per unit area is effective stress (σ_e). P is the pressure of water (pore pressure).

Relationship between σ_T, σ_e, and P is understood by using the same water-spring model used in slide 46.





Balance of stresses and pressure:

$\sigma_T = \sigma_e + P \quad \rightarrow \quad \Delta\sigma_T = \Delta\sigma_e + \Delta P \quad \rightarrow \quad \boxed{\Delta\sigma_e = \Delta\sigma_T - \Delta P}$

A decrease in σ_e may occur even when $\Delta\sigma_T = 0$, if $\Delta P > 0$.

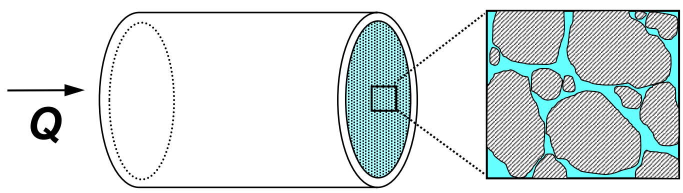
Springs get stretched → Weaken the solid-solid contact.

The strength of sediments decreases as σ_e decreases.
→ The effects of the rising water table on slope stability.

Solute Transport in Groundwater

Specific discharge and average linear velocity

Suppose Q ($m^3\ s^{-1}$) of water is flowing through a sand column.



Total area = A_{total}
Pore area = A_{void}
Porosity = $n = A_{void}/A_{total}$

Specific discharge q ($m\ s^{-1}$) =

Is this equal to flow velocity (v)?

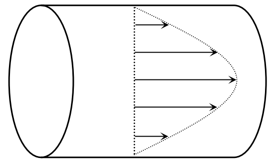
$q = (v \times A_{void}) / A_{total} = v \times A_{void}/A_{total}$

$\therefore q = nv$

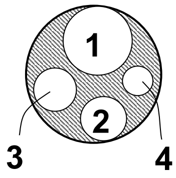
Note that $n < 1$ for most sediments and rocks. → $v > q$

Actual flow velocity varies within a pore tube.

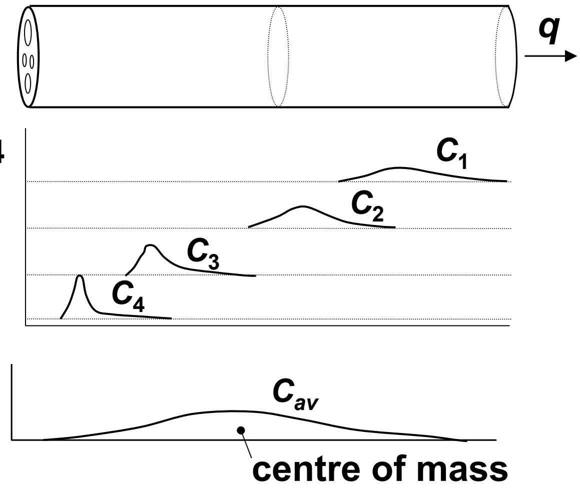
Why?



Flow velocity also varies among different-size pores.



Why?



Suppose that solute molecules are instantaneously released from the left end. Graphs show concentration 'profiles' in individual tubes at some time after the release.

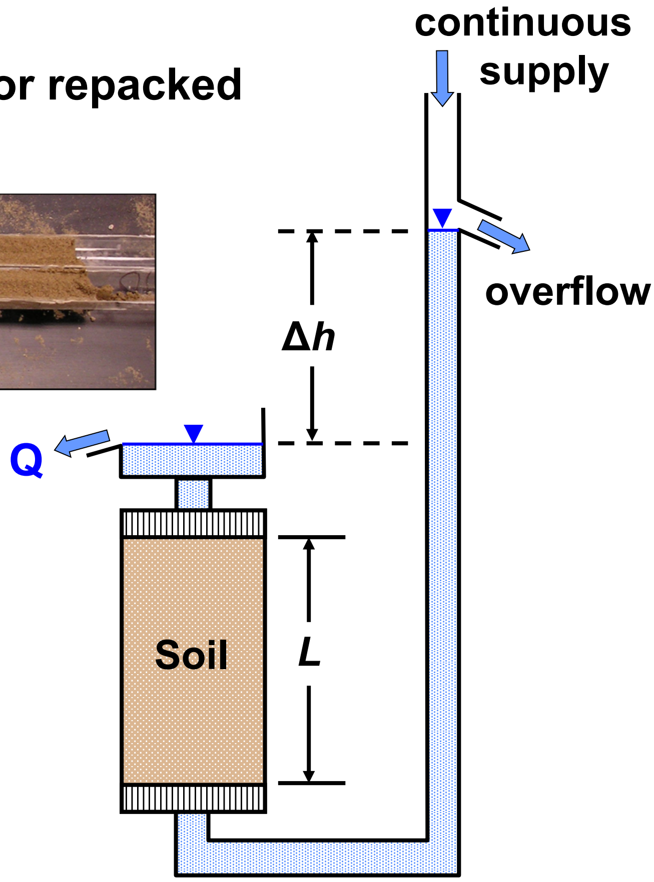
What we can 'measure' by sampling the column is the bottom profile, showing the average concentration of all pores. The centre of solute mass travel at the linear average velocity (v).

The spreading is called <u>dispersion</u>.

Hydraulic Conductivity Measurement

<u>Laboratory permeameter</u>

Measure the flow through intact or repacked soil samples.



Constant Head

$$\boxed{K = \frac{Q}{A}\frac{L}{\Delta h}}$$

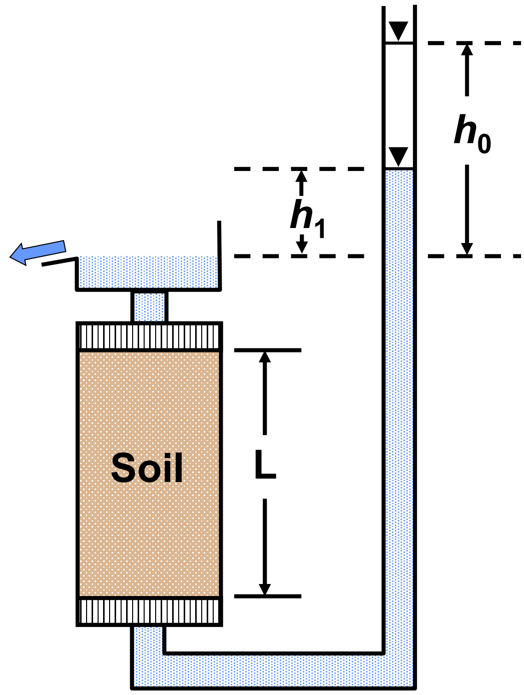
A = cross-sectional area of soil sample

Δh = head difference across the sample

Measuring the rate the water level falls in a riser tube is often easier and more reliable (especially for low K materials).

Falling Head

$$\boxed{K = \frac{a}{A}\frac{L}{(t_1 - t_0)}\ln\left(\frac{h_0}{h_1}\right)}$$



a **= cross-sectional area of riser tube**
A **= cross-sectional area of soil sample**
h_0 **= head difference across the sample at time t_0**
h_1 **= head difference across the sample at time t_1**

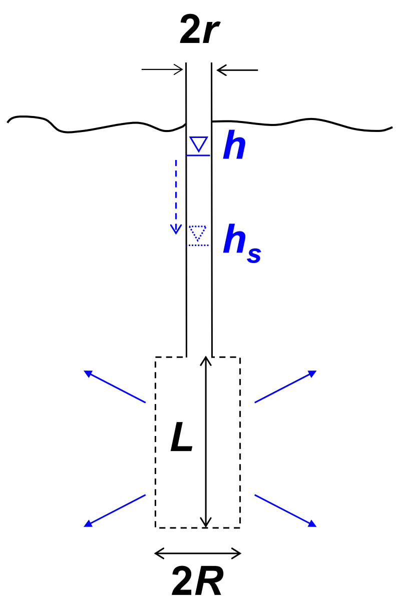
<u>Single-well response test (or slug test)</u>

Suppose the 'static' water level in a piezometer is h_s.

It is suddenly raised to the new level (*h*). What will happen to *h* over time?

The piezometer response is faster when:

-
-
-

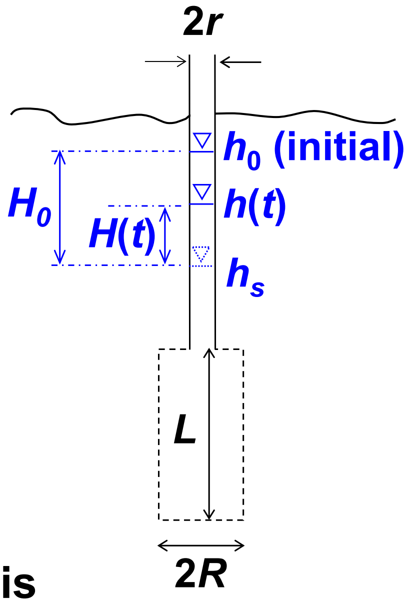


Physical model and equations

Initial displacement: $H_0 = h_0 - h_s$

Estimation of H_0 from slug volume (V_s):
$H_0 = V_s / (\pi r^2)$

Residual displacement: $H(t) = h(t) - h_s$



Zero-storage concept: At any given time, flow from (or to) the screen, Q (m^3/s) is:

$Q(t) = F \times K \times H(t)$

where F (m) is a shape factor and K (m/s) is the hydraulic conductivity.

What is F dependent on?

-
-
-

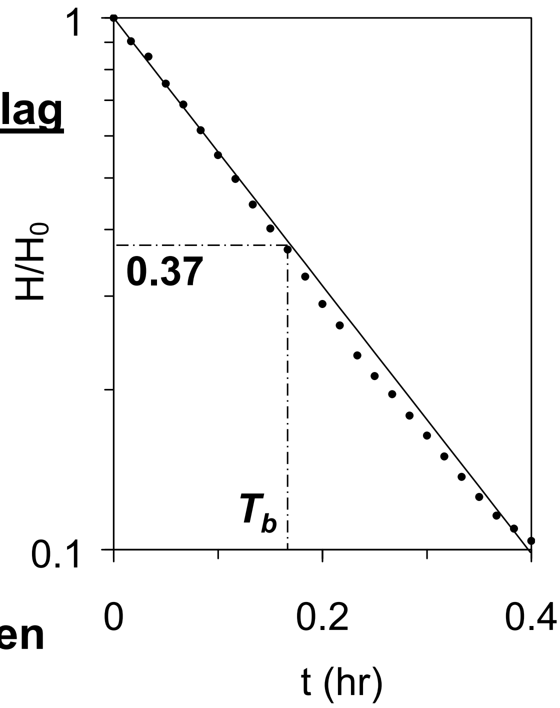
If the zero-storage model is reasonably good, then

$H(\mathrm{t}) = H_0 \times \exp[-FKt/(\pi r^2)]$ **exponential decay**

and we know $\exp(-1) \cong 0.37$.

$T_b = \pi r^2 / (FK)$ is called the <u>basic time lag</u> of the piezometer.

Or, $K = \pi r^2 / (FT_b)$ Eq. (1.6)



Now we need to estimate F.

Hvorslev (1951) equation:

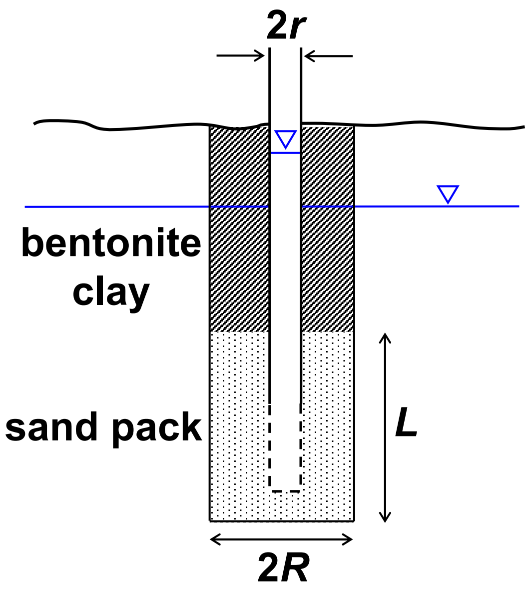
$F = 2\pi L / \ln(L/R)$ **Eq. (1.7)**

This is reasonable when:
- **No impermeable layer near the screen**
- **Water table is well above the screen**
- **$L/R > 4$**

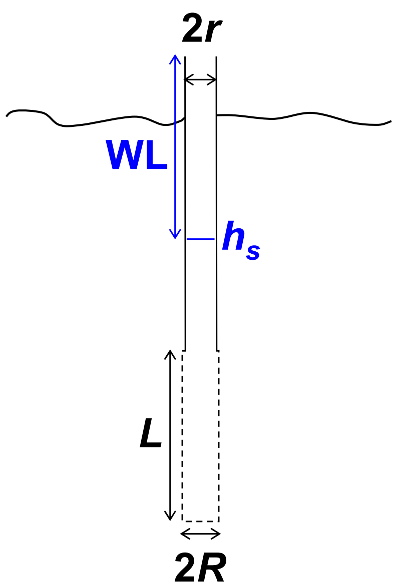
Sand Pack vs. Screen Size

In fine-grained materials, wells are often constructed with a sand filter pack around the well screen. The high-*K* sand pack will respond much quicker than the lower-*K* formation material.

In this case, the dimensions *L* and *R* should be taken from the sand pack, not the screen.



Example Problem No. 1



Time (min)	WL (m)	$h - h_s$ (m)	H/H_0
Static	2.400		
0	3.300		
1	3.120		
2	3.012		
3	2.860		
5	2.685		
8	2.578		
11	2.480		
15	2.446		
20	2.413		

$2r$ = 4.4 cm

$2R$ = 5.2 cm

L = 50 cm

K =?