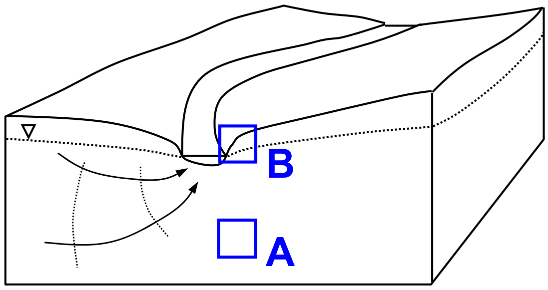
Lecture 2: Vadose Zone Processes

Darcy's law is useful in region A.

Some knowledge of soil physics is required to understand the processes in region B.

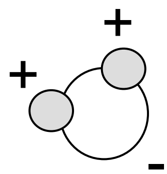


Important differences between A and B:

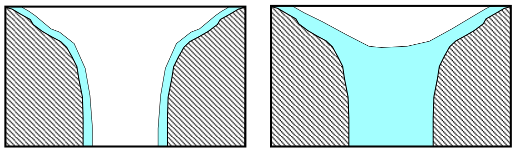
- Storage change is due to the compression/expansion of pore space in A. It is due to the filling/draining of pores in B.
- Hydraulic conductivity is dependent on water content in B.

Water storage in unsaturated soil

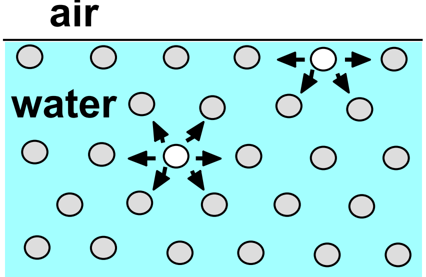
Mineral surfaces have uneven distribution of + and - charges, and they attract water molecules - hydrophilic.



Electrostatic attraction explains the storage of a thin film of water. The rest is held in soil pores by surface tension.

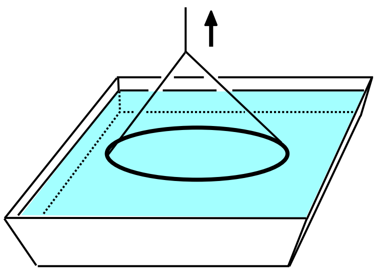


Molecules near the air-water interface feel stronger force inward than outward. A body of water tends to have the minimum surface area for a given volume.



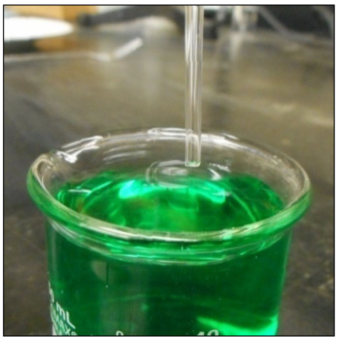
One needs to apply some force to increase the surface area of air-water interface. This force is called surface tension.

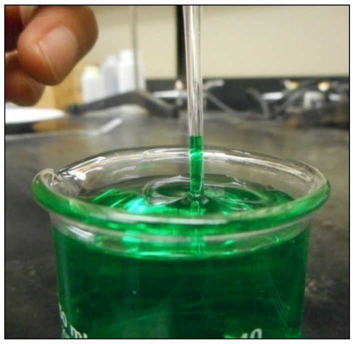
Unit of surface tension?



Capillary tube

The condition of water in soil pores is similar to water in a capillary tube (thin glass tube).

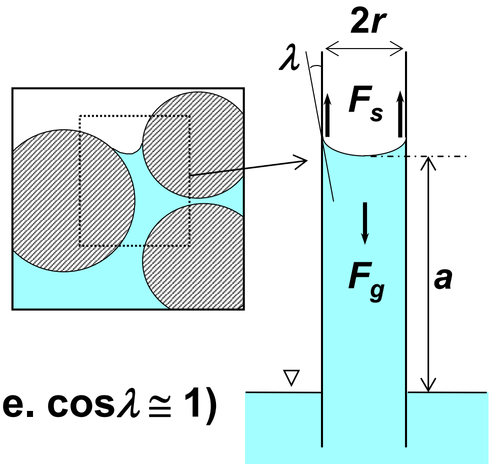




From the balance of downward force F_g (gravity) and upward force F_s (surface tension 'pull'), it can be shown that:

$$a = \frac{2\sigma}{\rho g}\frac{1}{r} \times \cos\lambda$$

Young-Laplace equation



ρ : density of water (1000 kg m^{-3})
σ : surface tension ($\cong$ 0.07 N m^{-1} at 20 °C)
λ : contact angle ($\cong$ 0 for most minerals; i.e. $\cos\lambda \cong 1$)

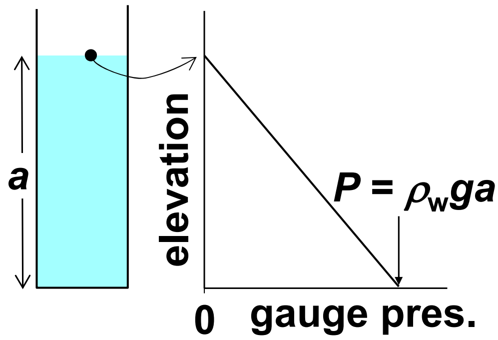
Example:

Estimate the height of capillary rise (*a*) in hypothetical mineral soil having a pore radius (*r*) of 0.1 mm.

Concept of negative pressure

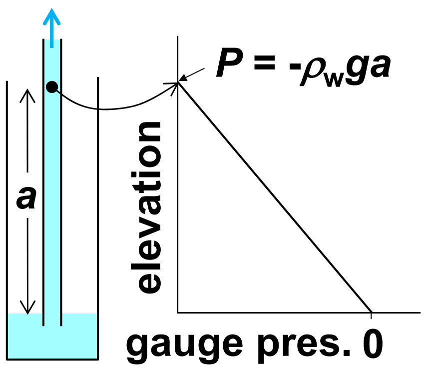
Gauge pressure is used in hydrology, which is referenced to atmospheric pressure.

P = 0 at the water surface in a glass and increases linearly with depth.



Now a straw is used to 'suck' water from the glass and keep it suspended.

Inside the straw, *P* increases with depth, but *P* = 0 at the bottom.
→ $P < 0$ in the straw!



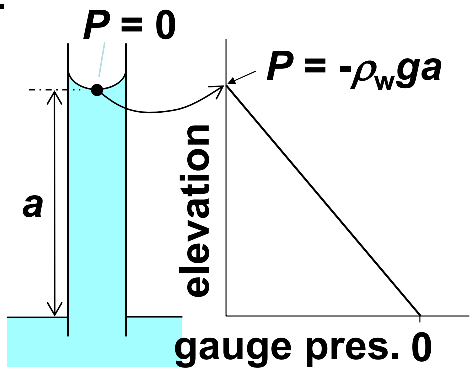
At a height *a* above the water surface in the glass,

$P = -\rho_w g a$ and $\psi = -a$

Negative pressure and pressure head.

Surface tension and negative pressure

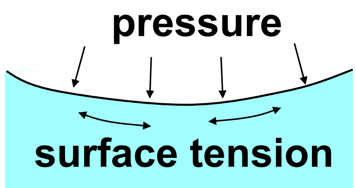
In a capillary tube, P also decreases with elevation.



Just below the air-water interface,
$P = -\rho_w ga$ and $\psi = -a$

Just above the interface,
$P = 0$ and $\psi = 0$

Abrupt change in P across the interface is similar to the pressure discontinuity in a soap bubble.

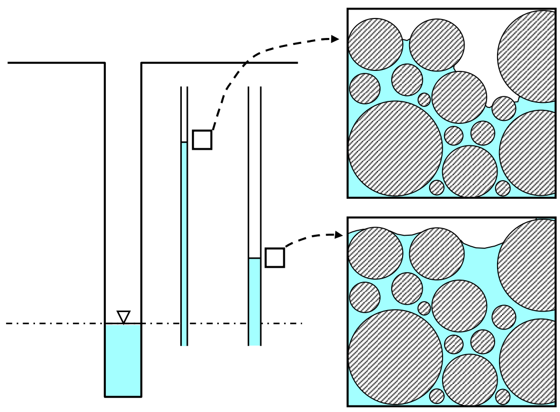


https://www.ntu.ac.uk/__data/assets/image/0026/1346813/bubble-for-web.jpg

Soil matric potential

In a similar manner, P and ψ in soils above the water table is negative. The magnitude of negative pressure is called soil tension. In soil physics, ψ is called soil matric potential head.

Soil particles are applying tension force to keep water suspended above the water table.



Under the hydrostatic condition (i.e. no flow of water), ψ is equal to the height above the water table.

Recall from Darcy's law:

$$h = z + \psi$$

$$\rho gh = \rho gz + \rho g\psi$$

The left-hand side is called total potential (J m^{-3}) in soil physics, consisting of gravity and matric potential. In saline soil, the effects of chemical osmosis needs to be added to total potential.

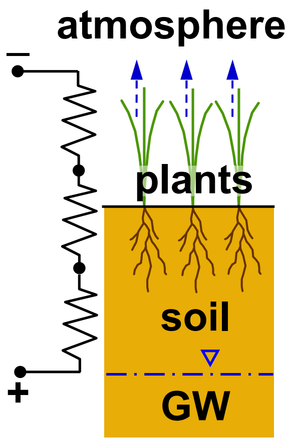
Soil matric potential represents the magnitude of energy (or work) required to remove water from soil pores.

→ Lower potential = Stronger attraction

The same matric potential is used by plant physiologists to represent the energy state of water in plant tissues.

→ Plants ‘suck’ water from soil, like straws.

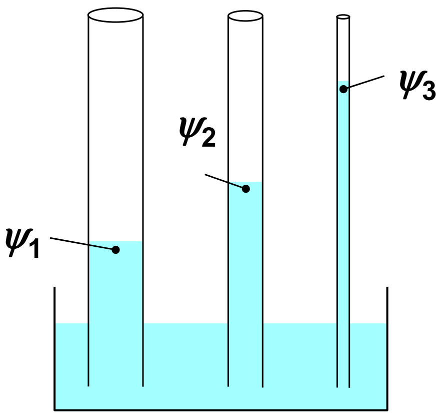
The matric potential is also related to vapour pressure in the atmosphere.



The flow of water through the <u>soil-plant-atmosphere continuum</u> is seamlessly described using ψ.

→ Basis for physically-based equations for estimating evapotranspiration (e.g., Penman-Monteith).

<u>Soil Water Characteristics</u>

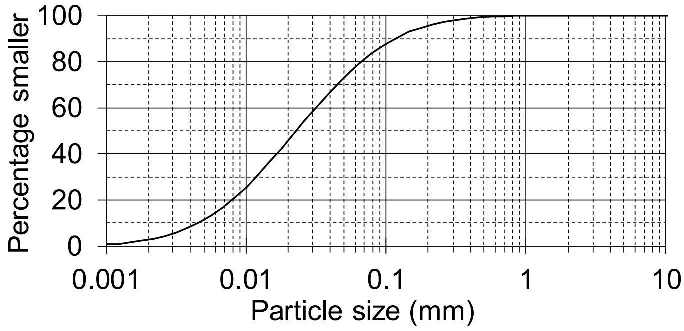


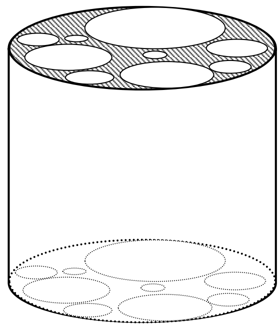
The height of capillary rise (and the magnitude of ψ) is related to the radius (r) of capillary tubes.

$$|\psi| \cong \frac{2\sigma}{\rho_w g}\frac{1}{r}$$

Smaller tubes have stronger ability to retain water against gravity.

Consider a bundle of different-size capillary tubes as a simplified model of soil pores.



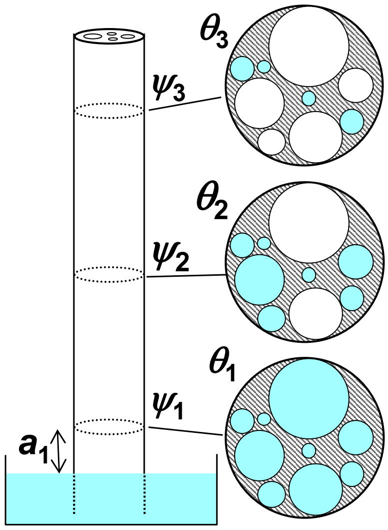


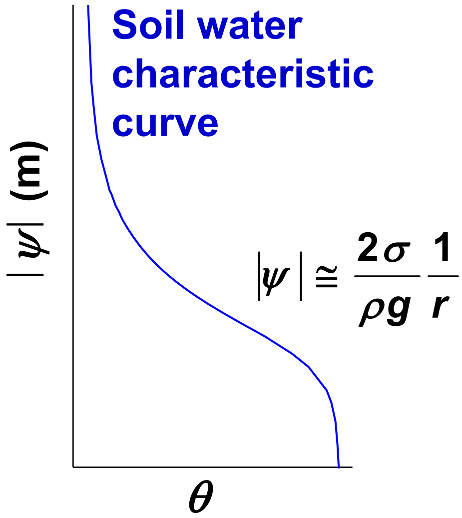
In each slice of the bundle, volumetric water content (θ) is defined by the sum of water-filled areas divided by total area.

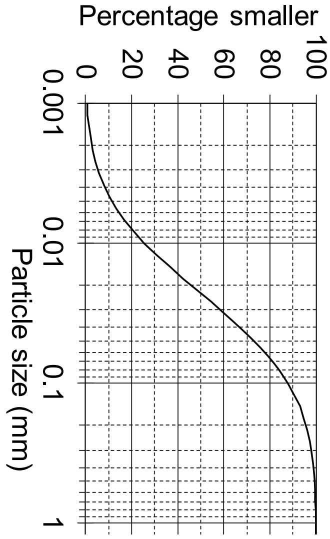
$$\theta = V_{water} / V_{total} \approx A_{water} / A_{total}$$

At Level 1 ($\psi_1 = -a_1$), the bundle is saturated because all tubes are holding water.

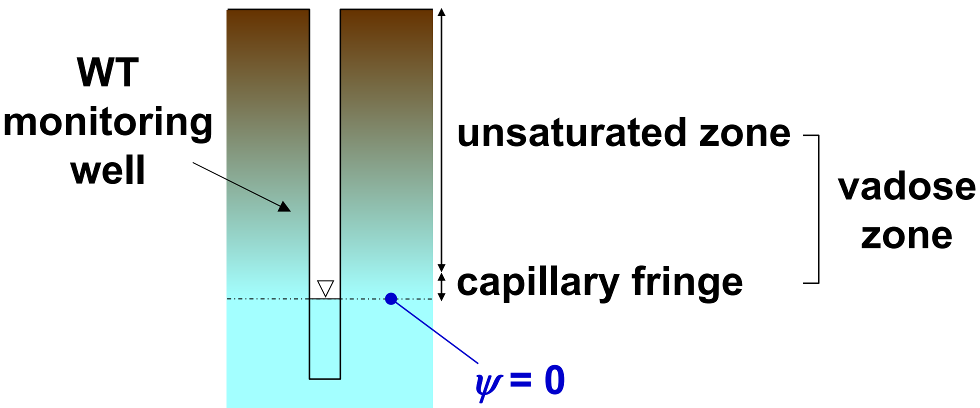
As large tubes become empty at some height above the water table, θ decreases with height.







In real soils under the hydrostatic condition (i.e. no flow of water), θ generally decreases with the height above the water table (WT).

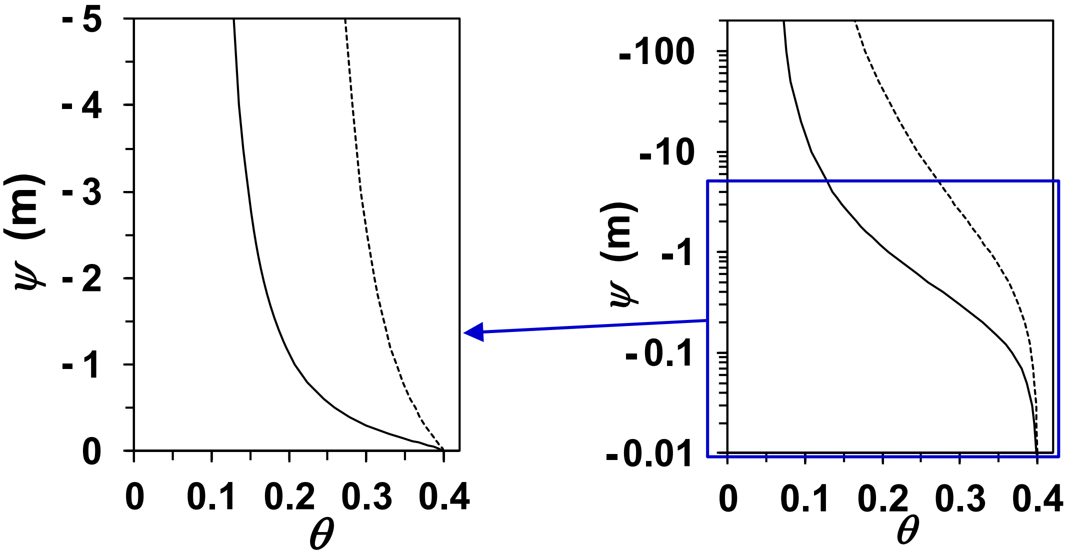


The saturated zone above the WT is called capillary fringe.

vadose zone = capillary fringe + unsaturated zone

Matric potential head is: negative in the vadose zone.
= 0 at the WT
positive below the WT.

Soil Water Characteristic (SWC) Curves



Which curve is for:
Sandy soil? Clayey soil?

Logarithmic plot of SWC
- Height of capillary fringe?

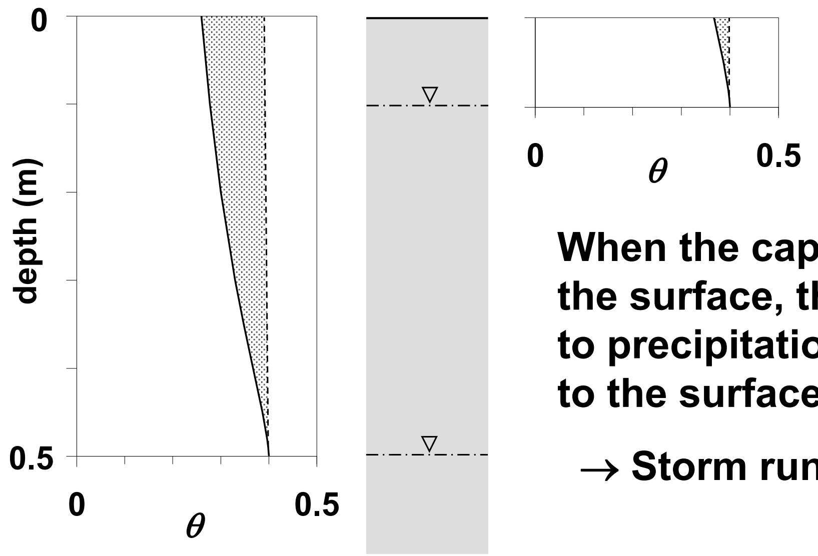
What does a very low value of ψ (e.g., -100 m) represent?
- Water is present in very small pores only.
- Easily accessible water has been taken up by evapotranspiration.

Dynamic response of capillary fringe

For the sandy soil in the previous slide, suppose a hydrostatic condition with the WT 0.5 m below the surface.

A sizable amount of water is required to saturate the soil column.

For the same sandy soil, suppose that the WT is 0.1 m below the surface. Only a small amount of water addition is required to saturate the soil and bring the WT to the surface.



When the capillary fringe is close to the surface, the WT responds quickly to precipitation events and moves up to the surface.

→ Storm runoff generation.

Unsaturated hydraulic conductivity

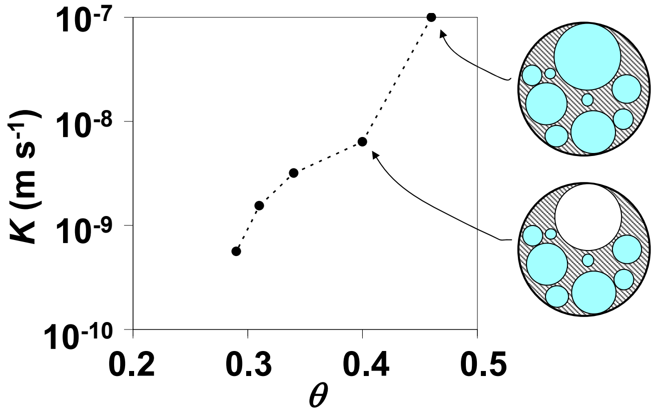
In the Darcy's law section, we saw that the hydraulic conductivity (*K*) of saturated sands is proportional to (pore diameter)^2.

We also saw that as the soil dries, average diameter of water-holding pores become smaller.

What does this mean?

The graph shows *K* as a function of θ for clay-rich soils in the Canadian prairies.
Hayashi et al. (1997. *Soil Sci.*, 162: 566)

$K(\theta)$ is highest at saturation and decreases with θ.



Effects of macropores

SW-GW interaction occurs mainly in shallow subsurface environments, where macropores (root holes, animal burrows, fractures, etc.) may provide the main conduits for water. $K(\theta)$ drops rapidly as the macropores drain.

Example:

Consider a root hole with a diameter of 2 mm. Is there water in this hole, if it is at 5 cm above the WT?

Richards equation

In the vadose zone: 1) Darcy's law needs to account for $K(\theta)$ function, and 2) storage is due to the change in θ. Therefore, the flow equation takes the form of:

$$\frac{\partial}{\partial x}\left(K_x(\theta)\frac{\partial h}{\partial x}\right)+\frac{\partial}{\partial z}\left(K_z(\theta)\frac{\partial h}{\partial z}\right)=\frac{\partial \theta}{\partial t}$$

The Richards equation plays the fundamental role in the analysis of SW-GW interaction involving the WT dynamics.

By solving the Richards equation, we try to determine θ at any time and space. However, K is dependent on θ, so we cannot solve the equation without knowing the solution first! This type of equation is called <u>non-linear</u>.

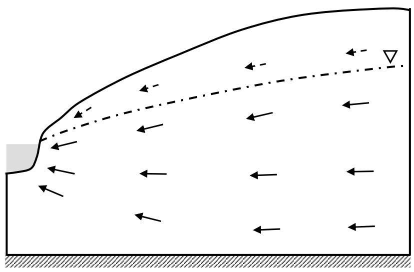
Non-linear equations are very difficult (or impossible) to solve by hand, and numerical solution on computers takes a very long time.

Therefore, simplified approach to obtain approximate answers is preferred in the studies of SW-GW interaction.

Simplified Approach for Estimating Discharge Flux

Approximation of hillslope flow with a 1D equation

<u>Dupuit-Forchheimer (D-F) approximation</u>



Suppose a vertical cross section with a stream. Actual flow field is two-dimensional involving the vadose zone.

D-F approximation assumes:

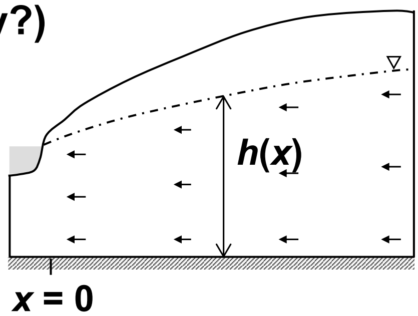
(1) Flow in the vadose zone is very small (why?)

(2) Flow is strictly horizontal.

(3) Hydraulic head (h) is a function of x only, meaning h does not change with depth.

(4) Aquifer has an impermeable bottom.

(5) Steady state (no change in the WT).

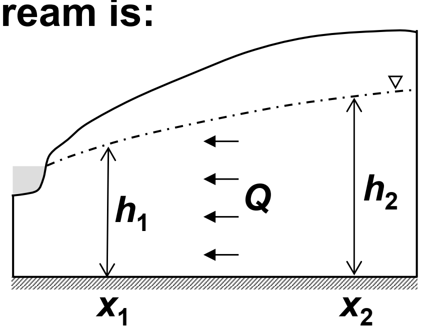


Remember that $h = z$ at the WT, so we can use the elevation of the WT as h. If we use the bottom of the aquifer as elevation datum, then h is numerically equal to saturated aquifer thickness.

Suppose that the section has a width (*y*-direction) of *w* (m). Then the flow rate *Q* ($m^3\ s^{-1}$) towards the stream is:

$$Q(x) = wh \times K_h \frac{dh}{dx} \qquad \text{Eq. (2.1)}$$

K_h ($m\ s^{-1}$) is saturated conductivity



To simplify the problem, we assume no recharge to the WT. Then *Q* is constant.

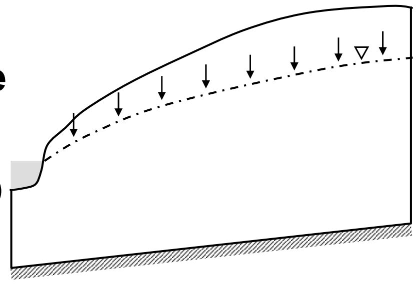
Solving Eq. (2.1) for constant *Q* with $h(x_1) = h_1$ and $h(x_2) = h_2$

$$Q = wK_h \frac{h_2^2 - h_1^2}{2(x_2 - x_1)} \qquad \text{Eq. (2.2)}$$

This can be also written: $Q = w \frac{h_2 + h_1}{2} \times K_h \frac{h_2 - h_1}{x_2 - x_1}$

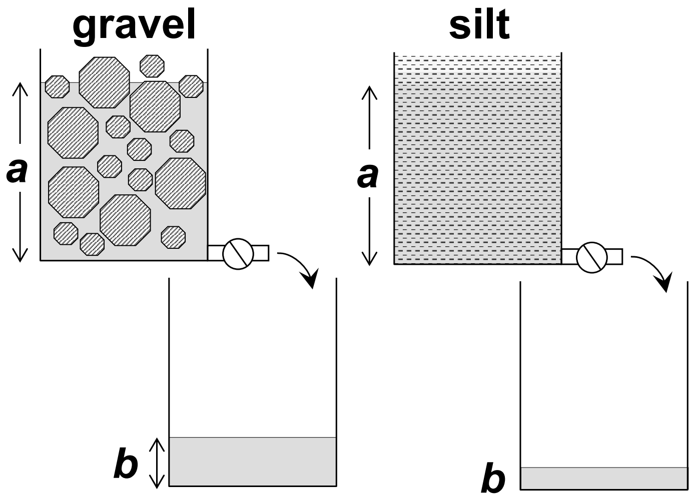
What is this?

The D-F approach is versatile and can include recharge and sloping boundary.



Dingman (2015. *Physical Hydrology*, Waveland Press, p.496)
Brutsaert (2005. *Hydrology*, Cambridge U. Press, p.388)

Specific yield of unconfined aquifer (revisited)



$$\frac{b}{a} = \frac{\text{drainage per area}}{\text{water table drop}}$$

→ Specific yield (S_y)

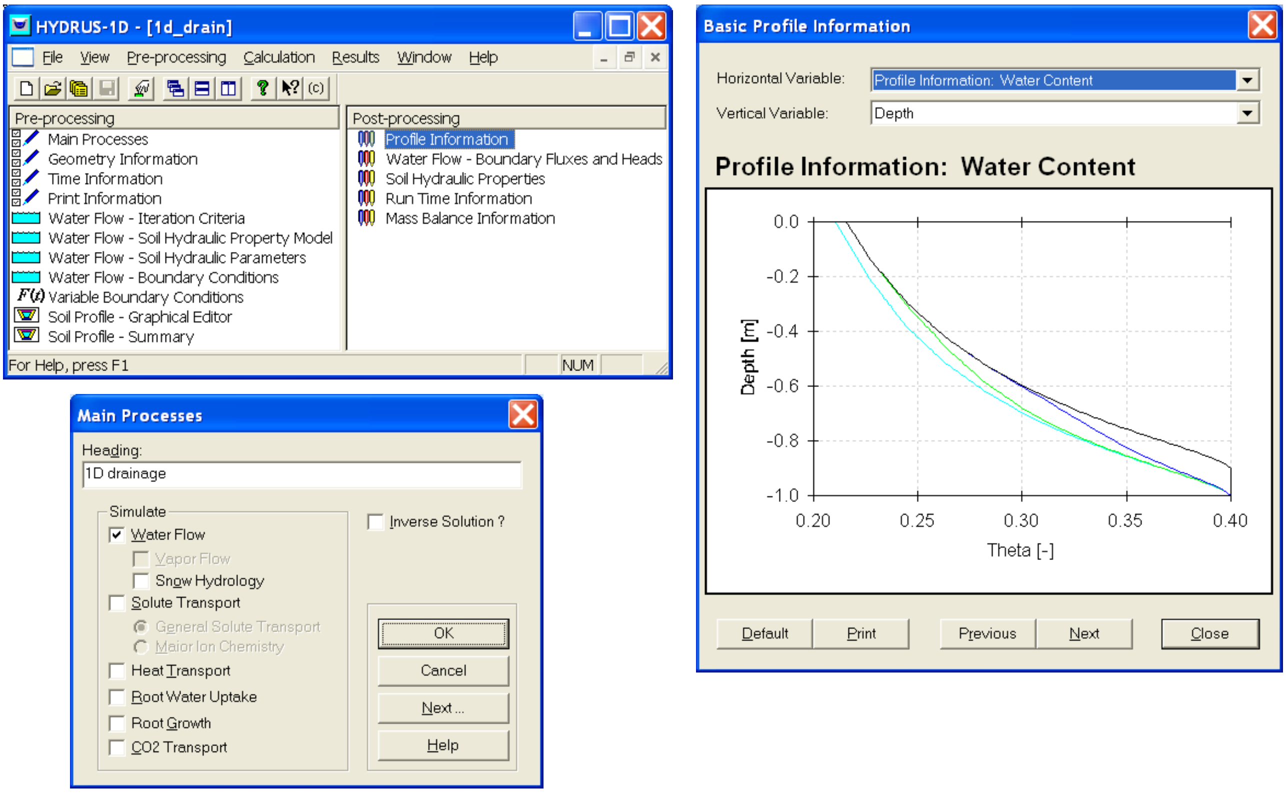
For gavel and coarse sands
$S_y \cong$ porosity

For finer size material
S_y << porosity

Above definition of S_y assumes that:
(1) Draining or filling of pores is instantaneous.
(2) Ratio *b*/*a* is independent of the depth to the water table.

Are these assumptions valid?

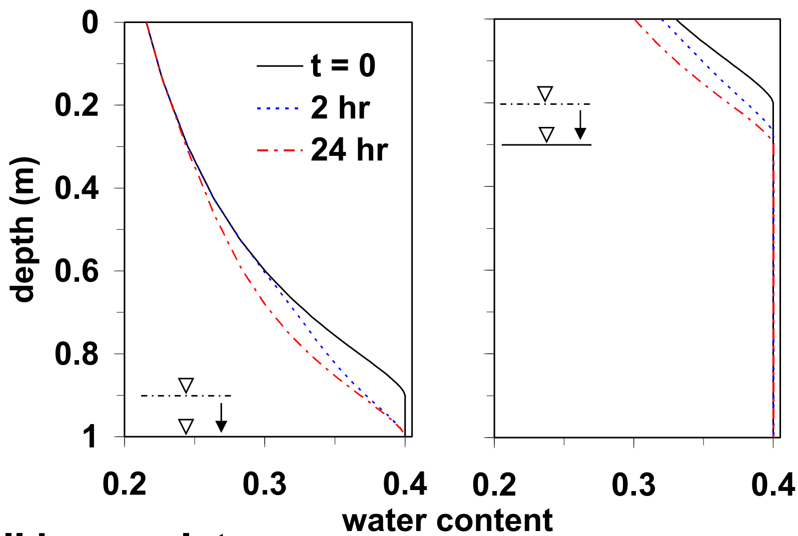
Drainage problems can be analyzed using numerical solutions of the Richards equation. → e.g., Hydrus-1D program



Consider the sandy soil from slide 11.

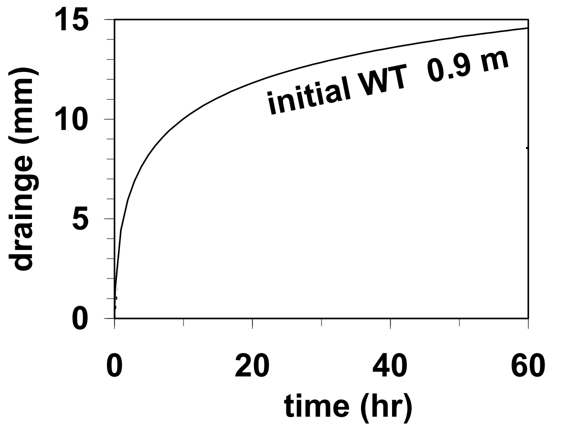
The WT is initially located 0.9 m below the surface (left) and lowered to 1.0 m at *t* = 0.

θ-depth profiles gradually change with drainage.



Note that the drainage is still incomplete at 24 hr. Using the value at 60 hr,

S_y =



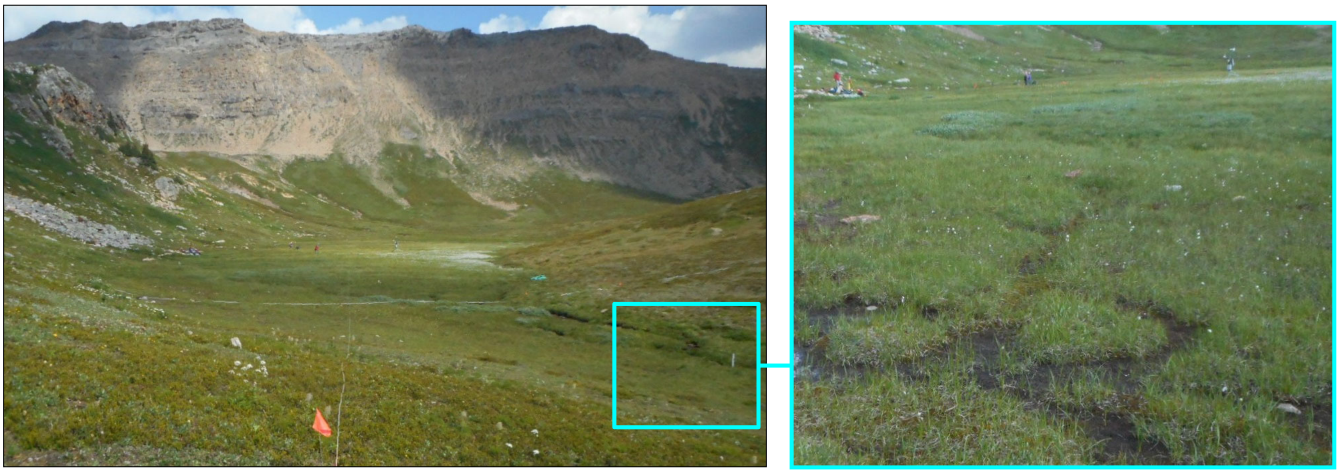
In the next example (top right), the WT is lowered from 0.2 m to 0.3 m.

The drainage completes at 20 hr,

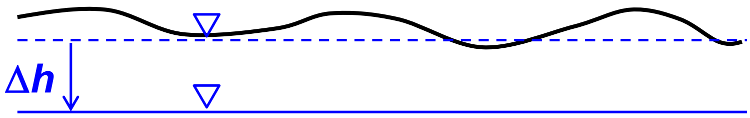
S_y =

Implications?

Water balance of wetlands



Helen Creek watershed, Canada

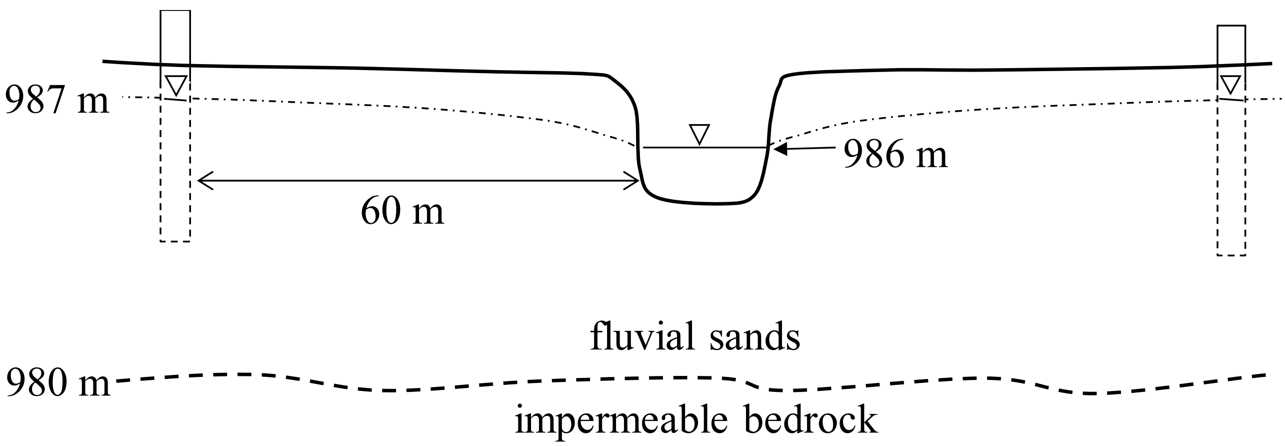


Volume change = Area × S_y × Δh

How do we define S_y? Is it constant?

See Sumner (2007. *Wetlands*, 27: 693-701)

Example Problem No. 2



Total flow (right and left) into a 500-m long stream section?