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Poems, Pulses and Polygons: How Classical Arabic Poetry Resonates with Music and Geometry

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Abstract

This paper investigates the geometric patterns inherent in Arabic poetry rhythms, uncovering attributes that likely contribute to their historical popularity. Through comprehensive analysis, we identified a combination of computational geometric features that seem to correlate with defining a rhythm's attraction in Arabic poetry. These features can be used to generate similar but new appealing poetry rhythms. While there are multiple studies on the geometry of musical rhythms, our research offers a novel perspective by applying these geometric concepts to poetry rhythms, aiming to contribute to the understanding of rhythmic patterns across diverse creative domains. To the best of our knowledge, our work is the first attempt to transfer the geometric analysis of rhythms to Arabic poetry rhythms. Moreover, our findings hint at potential connections between the rhythms of Arabic poetry and English song lyrics.

Keywords: Arabic Poetry; Musical Rhythms; Poetry Rhythms; Computational Geometry; Creativity; Computational Linguistics

1. Introduction

Poetry is a linguistic artifact and holds a prominent position in the literature and historical narratives of diverse cultures. While recurring rhythms may not be essential in some forms of poetry, they form a fundamental characteristic of classical Arabic poetry which is our main focus in this study. The inherent connection between poetry and music has long been recognized, evident in the shared terminology such as "rhythm". However, limited comparative studies have been conducted on the rhythms of music and language. This scarcity can mainly be attributed to the complexity of the involvement of experts in each respective domain [13]. In this paper, we aim to explore the parallels between the rhythmic percussion perceived in classical Arabic poetry and music, utilizing a computational geometric perspective.

Classical Arabic poetry rhythms have a historical connection to drum beats, a relationship documented in numerous prosody (*ʿArūḍ*) texts. The earliest can be traced back to *ʿAbdullah ibn al-Muʿtazz* (†c. 908 CE) [3], who narrated that *al-Khalīl al-Farāhīdī* was inspired by the sound pattern produced by the pounding hammers when he was passing by the street of braziers in *al-Baṣrah*, which prompted him to establish the foundations of *ʿArūḍ* basing the scansion on these patterns. Interestingly, a similar narrative exists in the account of Nicomachus of Gerasa [12], who described

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how Pythagoras discovered the diatonic scale. Remarkably, both findings also exhibit similar geometric features, as we will further elaborate.

This study looks at the rhythmic patterns in classical Arabic poetry from a computational geometric perspective, comparing them to traditional music rhythms, and possibly even to English song lyrics. Furthermore, we examine the inter-class similarities revealing a possible correlation with why certain rhythms are used more often than others. We also discovered that these classical Arabic rhythms have specific characteristics such as being evenly spaced, palindromic, and symmetric. Understanding these patterns could help create new unique and appealing rhythms that can be used in Arabic poems and transferred and tested for other languages. We summarize the paper's contributions in the following:

- Expanding the application domain of computational geometric analysis from musical to speech rhythms.
- Providing a comprehensive analysis that uncovers geometric patterns in classical Arabic poetry rhythms, which likely play a role in their historical popularity and aesthetic appeal.
- Presenting an algorithmic approach for generating new poetry rhythms that adhere to prosodic constraints yet offer potential for creative exploration beyond the standardized forms.
- Drawing parallels between Arabic poetry rhythms and musical rhythms, and possibly rhythms found in English song lyrics, opening up a new avenue for comparative studies.

2. Background

In this section, we provide a concise introduction to classical Arabic prosody, elucidating the employed meters and their respective popularity. Furthermore, we present the geometric representation of rhythms, accompanied by an exploration of some of its properties. Subsequently, we apply this geometric framework to the most frequently employed rhythms in classical Arabic poetry.

2.1. The Arabic Prosody and Rhythm Encoding

Al-Khalīl al-Farāhīdī, a renowned Arabic lexicographer and grammarian who lived during the period of 718 to 786 CE, is widely recognized as the first to establish the principles of Arabic prosody. Through a comprehensive analysis of existing Arabic poems during his time, *al-Khalīl* identified and analyzed fifteen distinct meters (*Buḥūr*). *Al-Khalīl* organized these patterns into five groups, each referred to as a "circle" due to their cyclic nature and because they share geometric features. One additional meter believed to be overlooked by *al-Khalīl* was later discovered by one of his students, *al-Akhfash*, but remained within the circles outlined by *al-Khalīl* [9].

In contrast to the syllable-based approach commonly used in Western prosody, the scansion method of *al-Khalīl* employs a mora-based system which is akin to musical rhythms, comprising distinct patterns of notes and rests. Specifically, this approach recognizes a mora as a single prosodic length [9]. A *ḥarf* in Arabic is the equivalent to a mora and can be: (1) Vocalized *ḥarf* (*mutaḥarrik* meaning movant), representing a consonant with a short vowel (notated as CV). This corresponds to a note in the rhythmic pattern. (2) Unvocalized *ḥarf* (*sākin* meaning quiescent), representing a consonant without a following vowel ($C\emptyset$), a second vowel in a long vowel formation, or a glide in a diphthong following a short vowel ($\emptyset V$). This is analogous to a pause or rest in the rhythm.

For clarity, we use the terms *note* for *mutaḥarrik* and *rest* for *sākin*. Consider the Arabic name ***Fāṭimah***. Its pronunciation percussion can be likened to striking a drum three times in a five-pulse interval, specifically at positions 1, 3 and 4. The scansion of ***Fā-ṭi-mah*** is as follows: $CVV - CV - CVC$, which can be encoded as $10 - 1 - 10$ where 1 signifies a note and 0 a rest. This can be expressed onomatopoeically as ***dum-da-dum***.

A full grapheme-to-phoneme conversion is unnecessary as Arabic script, with its diacritics, is generally moraic. Typically one mora corresponds to one grapheme [9]. However, it is necessary to exclude any silent grapheme and include any missing grapheme, like in cases of gemination (*tashdīd*), nunation (*tanwīn*), and saturation (*ishbāʿ*).

2.2. The Arabic Poetry Meters

A classical Arabic poem is structured in the form of couplet verses (2-hemistichs per verse), with each verse referred to as a *Bayt* and each hemistich as a *Shaṭr*. The meter is a periodic rhythm of notes and rests repeated across the poem.

بِذِي خَالِ	فِيَاتُ	لِسَلْمَى عَا	دِيَارٌ	Single Hemistich (Arabic)
مَفَاعِيلُن	فَعُولُن	مَفَاعِيلُن	فَعُولُن	Mnemonics (Arabic)
Mafāʿīlun	*Faʿūlun*	*Mafāʿīlun*	*Faʿūlun*	Mnemonics (English)
CVCVVCVVCVC	*CVCVVCVC*	*CVCVVCVVCVC*	*CVCVVCVC*	Consonants/Vowels
1101010	11010	1101010	11010	Encoding

Table 1: **Read from right to left.** A single hemistich from a love poem composed by *Umru al-Qays* (†c. 544 CE), employing the *Ṭawīl* meter. Hemistich translation: *"Abodes for Salmā effaced at Dhi Khal"*.

Isomorphic Group	Arabic Meter	Arabic Name	Standard Rhythm Pattern	Repetition
Al-Mukhtalif (Circle 1)	*Ṭawīl* *Madīd* *Basīt*	الطويل المديد البسيط	110 10 110 10 10 10 110 10 10 110 10 10 110 10 110	4 per verse
Al-Mu'talif (Circle 2)	*Kāmil* *Wāfir*	الكامل الوافر	1110 110 110 1110	6 per verse
Al-Mujtalab (Circle 3)	*Ramal* *Rajaz* *Hazaj*	الرمل الرجز الهزج	10 110 10 10 10 110 110 10 10	6 per verse
Al-Mushtabih (Circle 4)	*Sarīʿ* *Munsariḥ* *Khafīf* *Muḍarīʿ* *Muqtaḍib* *Mujtath*	السريع المنسرح الخفيف المضارع المقتضب المجتث	10 10 110 10 10 110 10 10 10 1 10 10 110 10 10 10 110 10 110 10 110 10 10 10 110 10 110 10 110 10 10 10 110 10 110 10 10 10 10 10 110 10 110 10 10 110 10 10 110 10 110 10 10 110 10	2 per verse
Al-Muttafiq (Circle 5)	*Mutaqārib* *Mutadārak*	المتقارب المتدارك	110 10 10 110	4 per verse

Table 2: The standardized form of Arabic meters within *al-Khalīl*'s circles.

Notably, the shortest periodic pattern is consistently repeated at least two times within each verse. For example, a complete couplet from the *Ṭawīl* meter is represented in its standard form by four repetitions of the following mnemonic words: ***Faʿūlun Mafāʿīlun*** which has the percussion pattern **da dum dum da dum dum dum** (See Table 1). This recurring periodic rhythm serves as a defining feature of an Arabic poem.

The Arabic poetry meters can be classified into two categories: the standardized forms, established by *al-Khalīl*, and the "de facto" forms which are considered by *al-Khalīl* to be derived from the standardized form. It is important to note that some of the standardized forms were rarely utilized in practice or were only used in clipped versions. Furthermore, deviations from the standardized forms were common but *al-Khalīl* introduced derivation rules that encompassed all of these deviations. Table 2 presents the standardized forms for each one of the sixteen meters. While *al-Khalīl*'s prosody encompasses additional complexities beyond the scope of this study, our focus in this research is solely on the rhythmic pattern aspect, focusing on the standardized forms.

2.3. The Poetry Meter's Popularity

Al-Khalīl sought to include in his study even the rhythms that appeared infrequently, albeit with a few instances of usage. Nonetheless, certain meters received greater prominence compared to others in terms of frequency of usage.

In order to gain insights into their prevalence, we examine the statistical data derived from the corpus compiled by the Department of Culture and Tourism in Abu Dhabi, UAE [1]. We chose only the poems that were written before the establishment of Arabic prosody. We believe that this would decrease the effect of the systematic copying of rhythms due to the absence of the prosodic tools.

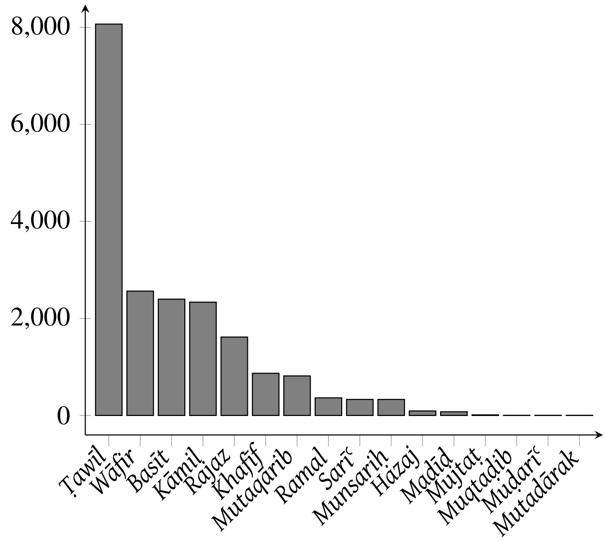

Fig. 1: Number of poems per meter during the pre-Islamic, early-Islamic, Umayyad, and early-Abbasid periods.

Figure 1 shows the historical frequency of usage of various Arabic meters preceding the discovery of Arabic prosody. It shows that the *Ṭawīl* meter stands out as the most widely used rhythm, surpassing the popularity of all other meters. The *Wāfir, Basīṭ, and Kāmil* occupy the second position, with closely matching frequencies. Following these, in the third position, is the *Rajaz* meter. The remaining meters were infrequently used, with some having only a few recorded instances.

In the next sections, we explore the geometric representation of rhythms, investigating the attributes inherent in the most renowned meters. Our goal is to shed light on the possible correlation with why such rhythmic patterns were preferred, thereby explaining the significantly higher frequency of usage of the *Ṭawīl* meter.

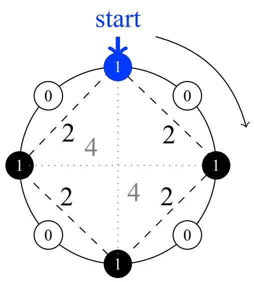

Fig. 2: Geometric representation of 10101010.

2.4. Geometric Representation of Musical Rhythms

Toussaint et al. [17] presented a geometric representation of cyclic musical rhythms by setting the notes and rests as vertices on a circle clockwise starting with the initial note and connecting consecutive note vertices with edges to form a convex polygon. Each pulse, whether denoting a note or a rest, corresponds to a vertex on the circle. By

[1] The statistics were collected using the filtering feature at https://poetry.dctabudhabi.ae/#/poems

connecting the notes in the circle, a convex polygon is formed. Figure 2 demonstrates this geometric representation for the periodic pattern 10101010. The starting note is marked in blue, and the rhythm plays clockwise. The edges are assigned weights that correspond to the geodesic distance between consecutive notes. The distance between two notes i and j is the sum of all vertices in between plus one. It is important to note that the geodesic distance represents the shortest path between any two notes, as an alternative shorter path may be available when traversing in the opposite direction (i.e., counterclockwise) around the circle. Consequently, the distance cannot exceed $\lfloor \frac{N}{2} \rfloor$, where N denotes the total number of vertices. The clockwise sequence of distances between notes is called the inter-onset interval (IOI) and it is in this case $I = 2222$.

3. Geometric Patterns in Arabic Rhythms

Arabic poetry rhythms can be encoded in a manner similar to music rhythms. As previously mentioned, Arabic poetry rhythms exhibit cyclic patterns of notes and rests, making them adequate to geometrical representation. Notably, *al-Khalīl* employed circles and vertices to illustrate the cyclic nature of poetry meters and used geometric isomorphism to group them.

3.1. Isomorphism

Isomorphism pertains to the concept where distinct rhythms are performed using a shared underlying arrangement of notes and rests. In formal terms, two rhythms, denoted as r_1 and r_2, are considered isomorphic if they possess an identical structure such that an overlap occurs if r_1 is rotated left or right, within a range of $1 < n < \frac{N}{2}$ times, to align with r_2. This engenders a sense of rhythmic diversity while upholding a uniform foundational pattern. *Al-Khalīl*'s study revealed that the rhythms present in Arabic poetry can be visually organized in a cyclic manner, and as a result, certain rhythms exhibit isomorphism. Consequently, he introduced the concept of circles (*Dawā'ir*) and categorized the 15 metrical patterns into five distinct circles: ***al-Mukhtalif***, ***al-Mu'talif***, ***al-Mujtalab***, ***al-Mushtabih***, and ***al-Muttafiq***. Each of these circles can be linearly unfolded into a one-dimensional rhythmic pattern, originating from a specific point, resulting in one of the 15 standard meters (see Table 2). This abstract theoretical framework not only facilitated the analysis of existing meters, but also enabled the generation of original isomorphic meters previously unexplored, which he terms *al-Buḥūr al-Muhmalah* or the neglected meters.

Analyzing the graph in Figure 1, it becomes evident that the majority of poems are composed using meters from the first three groups (88%). According to Frolov [9], the last two groups can be regarded as peripheral, housing anomalous and fictional patterns. It is noteworthy that the dataset from which we derived our statistical insights does not contain instances of *Muqtaḍib*, *Muḍarī'*, and *Mutadārak* from the early period, which signals their extremely limited usage then. Consequently, our main emphasis in this paper is on the first three groups, while the geometric attributes of the latter two groups will be mentioned later in the summary.

Al-Mukhtalif Isomorphic Group:. includes the most used rhythmic pattern, namely the *Ṭawīl* meter. As illustrated in Figure 3, the circle of the *Ṭawīl* meter is depicted alongside its isomorphic companions, *Basīt* and *Madīd*. Remarkably, this group incorporates ternary rhythms, characterized by their composition of 12 pulse intervals which is very prominent within traditional music trends [16]. Evidently, by executing a clockwise shift of the polygon in the *Ṭawīl* circle four times, it aligns with the *Basīt* polygon. Conversely, a counterclockwise shift of three positions results in the alignment with the *Madīd* polygon.

Al-Mu'talif Isomorphic Group:. includes two septuple rhythms characterized by the distribution of five notes over a seven-pulse interval and is constituted of only two meters: *Kāmil* and *Wāfir* in its standard form (refer to Figure 4).

Al-Mujtalab Isomorphic Group:. Similar to *al-Mu'talif*, this group is also septuple but with allocating four notes within the interval. Within this group lie the *Rajaz*, *Ramal*, and *Hazaj* meters in their standard forms (refer to Figure 5).

3.2. Symmetry and Palindromicity

A rhythm exhibits symmetry if it can be divided into two segments that mirror each other. Essentially, this means that folding the rhythm along a particular axis —whether vertical, horizontal, or diagonal— results in one side perfectly

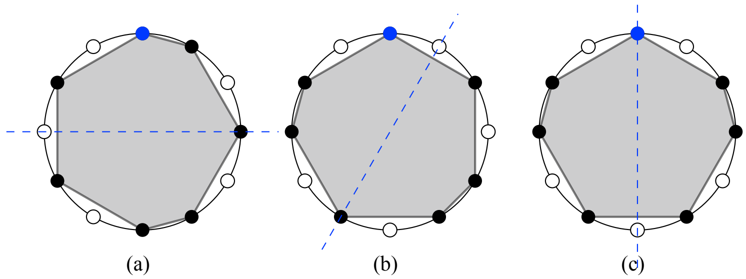

Fig. 3: *Al-Mukhtalif* Group: *Ṭawīl* (a), *Basīt* (b), and *Madīd* (c)

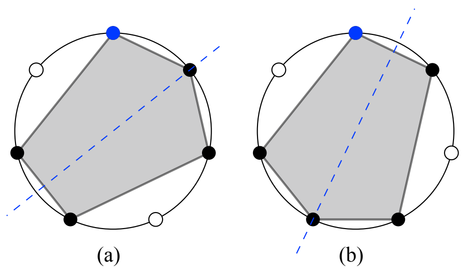

Fig. 4: *Al-Mu'talif* Group: *Kāmil* (a), and *Wāfir* (b)

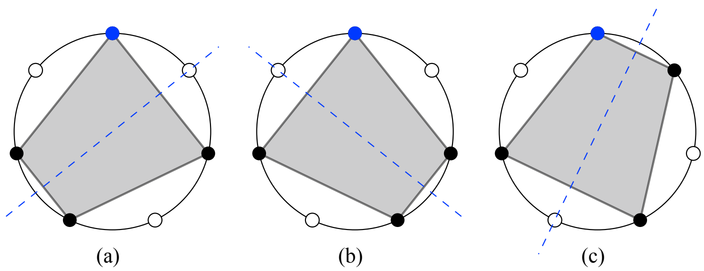

Fig. 5: *Al-Mujtalab* Group: *Rajaz* (a), *Ramal* (b), and *Hazaj* (c)

aligning with the other. In the case of vertical symmetry, where the axis traverses through the rhythm's initial note, the rhythm is characterized as palindromic.

A palindrome is a sequence that reads the same forwards and backwards. In rhythmic terms, this implies that the reversed form of a rhythm sequence replicates the original pattern. However, non-vertical symmetric rhythms sometimes permit being shifted to be played in reverse during a particular segment. Toussaint et al. [17] calls this variation a "weak" palindrome. Should one of the isomorphic rhythms display both symmetry and palindromic attributes, then all its corresponding patterns possess palindromic properties, although not necessarily "strong" palindromes. We can notice that all the meters within the three groups exhibit a type of symmetry. The line of symmetry is highlighted in blue in the figures. Nonetheless, only the *Madīd* meter, which is the least utilized meter in the group, demonstrates vertical symmetry and qualifies as a "strong" palindrome. The widely used *Ṭawīl* meter showcases horizontal symmetry, yet it is a "weak" palindrome. To transform *Ṭawīl* into a palindrome requires *three* counterclockwise shifts. In contrast, the *Basīt* meter presents a positive diagonal symmetry and requires *five* clockwise shifts to be palindromic (assuming a rhythm starts with a note).

Similarly, both *Kāmil* and *Wāfir* of *al-Mu'talif* group are "weak" palindromes and share with *Basīt* a positive diagonal line of symmetry. Unlike the previous two groups, the *Rajaz*, *Hazaj*, and *Ramal* meters exhibit symmetry; however, their symmetry lines do not pass through a note node. As a result, these meters are considered non-palindromic, implying that these rhythms cannot be played in reverse, regardless of the starting note.

3.3. The Deepness of Rhythms

Depth in rhythms pertains to the unique distribution of intervals between notes or rests.

Winograd-deep Rhythms:. A rhythm is Winograd-deep if every possible distance ranging from 1 to $\lfloor \frac{N}{2} \rfloor$ has a unique occurrence count (multiplicity), with N representing the total number of vertices. However, it is restrictive for rhythms necessitating the inclusion of exactly half of the potential distances [6].

Erdős-deep Rhythms:. A rhythm is Erdős-deep if all non-zero count distances have unique multiplicities. It only requires that the distinct distances with a non-zero count have a unique multiplicity [6].

The interval histograms showcased in Figure 6 outline the distribution of unique distances between notes within each of the three groups in study. Calculating all potential geodesic distances between notes, the histogram displays the frequency of occurrence for each unique distance. It is important to recognize that when calculating geodesic distances, the smallest non-zero distance is 1, and the largest is $\lfloor \frac{N}{2} \rfloor$.

Notably, isomorphic rhythms exhibit an identical interval histogram due to their identical distribution of distances as the distances between nodes remain unchanged, albeit in varying rotational orientations. For *Ṭawīl*, *Madīd*, and *Basīt*, which possess 12 vertices, the feasible geodesic distances are: 1, 2, 3, 4, 5, and 6. The interval histograms in figure 6-a demonstrate that each of these distances has a unique non-zero occurrence. Thus, this group adheres to Winograd and Erdős deepness. The rhythms within the second and third circles have 7 vertices, thus the possible distances are 1, 2, and 3. *Al-Mu'talif*'s histogram indicates that distances 1 and 2 share the same multiplicity, making these rhythms non-deep. Conversely, the histogram of *al-Mujtalab* displays unique multiplicities, making the rhythms Winograd and Erdős-deep.

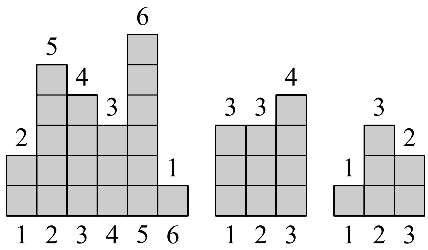

Fig. 6: *al-Mukhtalif* (a), *al-Mu'talif* (b), and *al-Mujtalab* (c) interval histograms

3.4. Evenness: Euclidean Rhythms

As previously explained, each rhythmic pattern consists of n notes distributed within a k-pulse interval, interspersed with $k - n$ rests. In this section we are interested in determining a distribution that spaces out n notes across a k-pulse interval as evenly as possible. This problem can be viewed as a subset of a broader problem involving the evenly distribution of n events in general across k positions, denoted as $E(n, k)$. This task mirrors the essence of Euclid's algorithm for determining the greatest common divisor of two numbers. This problem finds application in various domains, such as computing digital straight lines, determining leap years in calendar design, and even the timing systems in spallation neutron source accelerators, where pulses are evenly distributed across defined time events [6].

Within this context, Toussaint [15] applied the Bjorklund's algorithm (see Algorithm 1) to distribute n notes across a k-pulse interval to generate Euclidean rhythms. Additionally, Toussaint showed a relationship between Euclidean rhythms and Euclidian strings [7] where if one were to increase the first number in an inter-onset interval sequence I by one while decreasing the last number by one, the resulting sequence $\tau(I)$ becomes a rotation of the original. Toussaint [15] and Demaine et al. [6] extensively delved into rhythms originating from various cultural backgrounds, cataloging traditional world music that aligns with Euclidean rhythms and Euclidean strings. The prevalence of Euclidean rhythms and strings across diverse musical traditions became evident.

Returning to Arabic rhythms, *al-Mukhtalif* group has 7 notes distributed across a 12-pulse interval. Applying Bjorklund's algorithm with $E(7, 12)$ results in the binary string: 101101011010. While this rhythmic pattern holds potential

Algorithm 1 The Bjorklund Algorithm

```
function Bjorklund(group1, group2)
    if Len(group2) = 1 then
        return group1 + group2
    end if
    result ← empty list
    while Len(group1) and Len(group2) do
        result.append(group1.pop() + group2.pop())
    end while
    return Bjorklund(result, group1 + group2)
end function

function E(n, k)
    group1 ← [1]^n
    group2 ← [0]^(k−n)
    return Bjorklund(group1, group2)
end function
```

as a valid meter within Arabic poetry, it does not correspond to any of the three meters under examination. Notably, reflections, rotations, and complements of Euclidean rhythms retain their Euclidean nature [16]. Consequently, through rotation, it becomes possible to generate the three utilized meters from *al-Mukhtalif* group which implies that *Ṭawīl*, *Madīd*, and *Basīt* are all Euclidean rhythms. On the other hand, among the three meters, only *Ṭawīl* qualifies as a Euclidean string with an inter-onset interval of $I = 1221222$. Applying the previously described operation yields in $\tau(I) = 2221221$ which represents a right rotation of I by one position.

Al-Mu'talif group has a distribution of 5 notes across 7-pulse interval. $E(5, 7)$ yields 1011011, and the rotation generates both *Kāmil* and *Wāfir* meters. Similarly, *al-Mujtalab* group has a distribution of 4 notes across a 7-pulse interval. $E(4, 7)$ results in 1010101, which can be rotated to generate *Rajaz*, *Ramal* and *Hazaj*. Among these two groups, only *Kāmil* with two clockwise rotations, and *Hazaj* with one anti-clockwise rotation, are Euclidean strings.

3.5. Generating Arabic Euclidean Meters

By applying the Bjorklund algorithm in conjunction with rotation operations, we can generate rhythmic patterns that comply with the specific constraints of classical Arabic poetry. It is essential to outline these constraints [9], which are primarily influenced by the orthography of the Arabic language, as follows:

1. A verse cannot start with a *sākin* (rest) and must end with a *sākin*. $L_1 = \{0x \cup x1 \mid x \in \{0, 1\}^*\}$.
2. The occurrence of two consecutive *sākin*s is prohibited, except at a prosodic stop at the end of the verse. $L_2 = \{x00y \mid x \in \{0, 1\}^*, y \in \{0, 1\}^+\}$.
3. The sequence of consecutive *mutaḥarrik*s (notes) is limited to a maximum of four (*fāṣilah kubrā*), thereby prohibiting sequences of five or more consecutive *mutaḥarrik*s. $L_3 = \{x1^5y \mid x, y \in \{0, 1\}^*\}$.

From the constraints, we deduce that the set of valid rhythmic patterns is defined as $L_{valid} = \{0, 1\}^* \setminus (L_1 \cup L_2 \cup L_3)$. This formulation excludes patterns $E(0, k)$ and $E(k, k)$, since any rotation of such strings would infringe upon the first constraint. Patterns where $k > 2n + 1$ are considered invalid due to the inevitable presence of two consecutive zeros, which cannot be prevented through rotation. Furthermore, patterns where $\lfloor \frac{n}{k-n} \rfloor > 4$ are invalid, as they would necessitate five consecutive notes, a condition that cannot be rectified through rotation without ending the sequence with a note. If the ratio $\frac{n}{k}$ is reducible, then the rhythm possesses the same periodicity as that produced by the simplified ratio. Hence, $E(1, 2) = E(2, 4) = E(3, 6)$, and so on, since our focus is solely on the shortest repeating period.

Upon generating all possible valid Euclidean Arabic rhythms up to $k = 12$, we identified 45 valid patterns. Notably, at least 60% of these have been utilized in various extents within traditional poetry compositions. Among these, and in addition to the rhythms previously studied, only $E(3, 4)$ and $E(5, 8)$ exhibit horizontal symmetry due to their divisibility by 4. For instance, $E(3,4)$ generates a single valid rhythm, 1110, known as *Khabab* (or horse canter). We have counted at least 14 poems in the dataset with this pattern. Similarly, $E(5,8)$ yields three rhythms: 10110110, 11010110, and 11011010, with only the latter possessing both horizontal symmetry and being a Euclidean string. This particular pattern, identified as a version of *Mukhalla‘ al-Basīt*, is the most common among them in our dataset, appearing in at least 52 poems. From the first rhythm, we have counted only 4 poems, while we have encountered only a single instance of the second rhythm outside our dataset in a prosody book [1].

4. Cultural Resonance

The pattern 101011010101 is isomorphic to *al-Mukhtalif* group and can be found in traditional music. It corresponds to *Bembé*, an Afro-Cuban rhythm, and to the western diatonic scale pattern [6]. This exact rhythmic pattern was generated by *Ibn al-Qaṭṭā‘* (†c. 1121 CE) [4] as a neglected meter but with turning the final note after 4 repetitions into a rest to avoid the prohibited ending with a note.

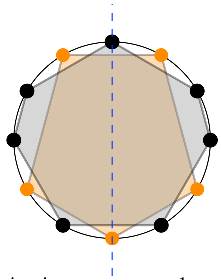

Fig. 7: *Al-Mu'talif* slow version in orange, complementing *al-Mukhtalif* in black.

Notably, *Fume-Fume*, a traditional Ghanaian rhythm with a pattern of 101010010100 [16], echoes the sound of *Kāmil* but with longer rest intervals between notes. This rhythm resembles a slower rendition of the *Kāmil* meter where a poem written on *Kāmil* is recited by lengthening short vowels and doubling the pause time on unvocalized *ḥarfs*, these practices can be noticed in the traditional Arabic songs such as in *al-Qudūd al-Ḥalabiya*, a form of traditional music from Aleppo that often incorporate songs from classical Arabic poetry [2]. Lengthening short vowels was also a known practice among early Arabs as described by *Ibn Jinnī* (†c. 1002 CE) [5] introducing examples from poetry and speech. If one rest is inserted between every two notes in the *Kāmil* circle, a ternary rhythm with 12 pulses similar to *al-Mukhtalif* group arises. Strikingly, this elongated rhythm perfectly complements *al-Mukhtalif* circle (Figure 7). This observation suggests the close affinity between *al-Mukhtalif* group, which contains the most used rhythm, and *al-Mu'talif* which contains the second most used rhythms, potentially explaining their widespread use in Arabic poetry literature. Moreover, unlike the original rhythm, the resulting elongated rhythm is Erdős-deep. Further exploration of rhythm similarities is planned for future investigation.

4.1. English Songs: A Provisional Exploration

Unlike the rhythms in Arabic poetry, which often rely on the sequence of moras, English poetry rhythms depend primarily on the alternation of stressed and unstressed syllables. In an early study, Allen [2] asked participants to tap their fingers in time with the beats they perceived in words. The results showed that these beats often coincide with the vowel onsets, a transition from consonants to vowels, and are more easily detected in stressed syllables. Revisiting this topic more recently, Rathcke et al. [14] came to a similar conclusion. This type of rhythm aligns with Arabic poetry rhythms, where scansion places rhythmic beats on vocalized consonants (those followed by short vowels).

Lerdahl [11] explores the connections between musical elements and syllabic poetry, using the Beatles' "Yesterday" as a key example. However, when we shift our attention from the emphasis on syllables to the inherent percussion produced by the sequence of consonants and vowels, we find recognizable patterns. Furthermore, unlike Arabic, English writing does not follow a "moraic" system, necessitating the translation of graphemes into phonemes to properly

[2] https://ich.unesco.org/en/RL/al-qudoud-al-halabiya-01578

sequence consonants and vowels. Additionally, It is necessary to address the presence of super heavy syllables (e.g., patterns like CVCC or CVVC) in English. We encode such syllables with 10 instead of the more expansive 100, simplifying rest clusters, often found in consonant clusters, to a single rest. A seminal study by Haggard [10] on a 70-word sample examined various types of consonant clusters in English. The findings indicated that consonants within clusters tended to have a shorter duration than those in non-clustered contexts, supporting the rationale for our proposed simplified encoding scheme.

Following this methodology, we can observe a familiar pattern in the refrains: "I believe in yesterday" and "Yesterday comes suddenly", coded as 101101010110 for the former and 101101011010 for the latter. The details can be found in Table 3. Intriguingly, these patterns are rotational variants of one another, and both fit into the *al-Mukhtalif* isomorphic group (Circle 1). The first matches the *Madīd* pattern, and the second corresponds to the result of the Bjorkland algorithm applied to $E(7, 12)$. Notably, both patterns exhibit traits of Euclidean rhythms and are characterized as being both Erdős-deep and Winograd-deep.

Through further analysis of the song's lyrics, we find the words "Yesterday" and "Suddenly" in the first lines of each stanza—encoded as 10110 and 11010 respectively. These are also rotational variations and can be classified under the *al-Muttafiq* isomorphic group. Both are Euclidean rhythms, as well as Erdős and Winograd-deep. Additionally, exploring lines from the first and second stanzas, respectively: "All my troubles seemed so far away" 1010110101010110 and "I am not half the man I used to be" 1010101101010110, we find these too are rotational equivalents, Euclidean, and deep. This finding may suggest cultural parallels in speech rhythms. We acknowledge that a deeper exploration into English lyrics remains an avenue for future research.

Phrases	I	believe	in	Yesterday	Yesterday	comes	suddenly
IPA	ʔaɪ	bɪˈli ːvɪn		ˈjes tədeɪ	ˈjes tədeɪ	kʌmz	ˈsʌdən liː
Consonants/Vowels	CVV	CVCVV CVC		CVCCVCVV	CVCCVCVV	CVCC	CVCVCCVV
Endoding	10	110 10		10110	10110	10Ø	11010
Percussion	dum	da-dum dum		dum da-dum	dum da-dum	dum-m	da-dum dum

Table 3: The scansion procedure for the refrains of the Beatles song *"Yesterday"*. The crossed-out zero is removed for simplifying rest clusters.

5. Discussion

Our findings, summarized in Table 4, suggest that while no single attribute solely defines a rhythm's popularity, a combination of features might play a role. For instance, the *Ṭawīl* rhythm uniquely combines several attributes such as ternary, symmetry, and palindromic qualities. Among standard rhythms, only *Ṭawīl* demonstrates horizontal symmetry. Its weak palindromic nature might contribute to its popularity, as both non-palindromic and perfectly palindromic traits seem less favored, exemplified by rhythms like *Madīd* and *Mutadārak*, which also correspond with vertical symmetry. While the ternary nature of *Ṭawīl* is distinctive, being ternary alone does not ensure popularity. Other rhythms in the *al-Mushtabih* group, although ternary, lack other attributes in study. On the other hand, the *Mutaqārib* rhythm is not ternary and has a more distant palindromic rotation (2 out of a possible 2 compared to *Ṭawīl*'s moderate rotation of 3 out of a possible 6), highlighting the significance of having a balance of the studied characteristics. In summary, while some individual rhythmic properties are essential, it appears that a balanced combination of certain attributes, like weak palindromicity, horizontal symmetry, and a moderate number of rotations, is important in defining the attractiveness of a rhythm. These findings resonate with previous research on traditional music [16].

6. Conclusion and Future Work

In this paper, we discussed interesting geometric patterns in the rhythms of Arabic poetry. Our investigation highlights that certain geometric attributes are present in appealing rhythms. Meters that strike a balance in properties such as palindromicity and symmetry tend to be preferred. A majority of the standard meters adhere to a maximally

Meters	*Ṭawīl*	*Basīt*	*Madīd*	*Kāmil*	*Wāfir*	*Rajaz*	*Ramal*	*Hazaj*	*Sarīʿ*	*Munsariḥ*	*Khafīf*	*Muḍāriʿ*	*Muqtaḍib*	*Mujtath*	*Mutaqārib*	*Mutadārak*
Ternary Rhythm	✓	✓	✓	×	×	×	×	×	✓	✓	✓	✓	✓	✓	×	×
Symmetry	H	D	V	D	D	D	D	D	×	×	×	×	×	×	D	V
Palindrome	W	W	✓	W	W	×	×	×	×	×	×	×	×	×	W	✓
Rotations to Palindrome	3/6	5/6	0/6	1/3	3/3	-	-	-	-	-	-	-	-	-	2/2	0/2
Erdős and Winograd-deep	✓	✓	✓	×	×	✓	✓	✓	×	×	×	×	×	×	✓	✓
Euclidean rhythm	✓	✓	✓	✓	✓	✓	✓	✓	×	×	×	×	×	×	✓	✓
Euclidean string	✓	×	×	✓	×	×	×	✓	×	×	×	×	×	×	✓	×
Complements	2*	2*	2*	1*	1*	×	×	×	×	×	×	×	×	×	×	×

Table 4: Findings Summary. (H) indicates horizontal, (D) diagonal and (V) vertical symmetries. (W) indicates weak palindrome and (*) indicates that the comparison is with the slow version of circle 2.

even (Euclidean) structure, with at least half being deep rhythms. The *Ṭawīl* meter, in particular, stands out due to its unique and balanced combination of several characteristics. This echoes a broader theme across creative domains where appreciation is correlated with balance [8]. Our findings show an overlap with traditional musical patterns and raise questions about possible parallels with English song lyrics.

Our work extends our understanding of what types of aesthetic qualities resonate with people. Yet, it is important to acknowledge some limitations. Our analysis is based on the few standardized rhythms used in the Arabic literature, which may not cover the full variety. Future endeavors should address the variations and deviations from the standardized forms within the broader corpus of Arabic poetry. In addition, we intend to delve deeper into the parallels between poetry rhythms and traditional music, exploring how these might be used in creative applications, like composing songs over musical beats. A promising direction is the generation of new poetry rhythms, capitalizing on our findings, that can be verified through human evaluations.

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